

160) Determinate i seguenti insiemi:

$$i) A = \{x \in \mathbb{R} : \sin x = \frac{1}{2}\}$$

$$vi) A = \{x \in \mathbb{R} : \frac{x^2 - 2x}{x^2 - 4x + 3} \leq 0\}$$

$$ii) A = \{x \in \mathbb{R} : \sqrt{x+2} = \sqrt{4-x}\}$$

$$v) A = \{x \in \mathbb{R} : \sqrt{x^2 - 4} \geq x\}$$

$$iii) A = \{x \in \mathbb{R} : \sqrt{-x} = -2\}$$

$$vii) A = \{x \in \mathbb{R} : e^{x^2 - 3x + 2} \leq 1\}$$

$$iv) A = \{x \in \mathbb{R} : 2x^3 + 3x^2 - 2x - 3 \geq 0\} \quad viii) A = \{x \in \mathbb{R} : \log(3x - 2x^2) < 0\}$$

161) Determinate gli insiemi

$$i) A = \{x \in \mathbb{R} : 5x + 3 = -7x - 3 \quad o \quad 2x^2 - 3x - \frac{35}{9} = 0\}$$

$$ii) A = \{x \in \mathbb{R} : \frac{x-1}{x-3} \geq x\}$$

$$iii) A = \{x \in \mathbb{R} : \frac{x^2+1}{x+1} < 2x-3\}$$

$$iv) A = \{x \in \mathbb{R} : \sqrt{1-x^2} \leq 2x\}$$

$$v) A = \{x \in \mathbb{R} : \frac{\sqrt{x}}{\sqrt{x}-1} < \sqrt{x} + 1\}$$

$$vi) A = \{x \in \mathbb{R} : \sqrt{x} \leq 2x \quad o \quad \frac{x-1}{x+3} > 4\}$$

$$vii) A = \{x \in \mathbb{R} : 2\sin^2 x - 5\sin x + 2 < 0\}$$

$$viii) A = \{x \in \mathbb{R} : 2\cos^2 x + 3\sin x - 3 \leq 0, 0 \leq x \leq \frac{5}{2}\pi\}$$

$$ix) A = \{x \in \mathbb{R} : x^2 + 4x \geq 0, x^2 + 4x < \left(\frac{1}{5}\right)^{-1}\}$$

$$x) A = \{x \in \mathbb{R} : \sqrt{x^2 - x - 12} \cdot \log(x+6) \geq 0\}$$

$$xi) A = \{x \in \mathbb{R}: \log(3x+5) - \log(6x+1) \leq \log 3\}$$

$$xii) A = \{x \in \mathbb{R}: \log x \geq -1, 3^x + 3^{x+1} + 3^{x+2} \leq 39\}$$

$$xiii) A = \{x \in \mathbb{R}: 2e^{\frac{x^2}{2} + x - 1} \cdot e^{-x-3} \leq 2, 2^x + 2^{x+1} - 2^{x+2} + 2^{x+3} \geq 7\}$$

$$A = \{x \in \mathbb{R}: \cos^2 x - \sin^2 x = \frac{1}{2}, 0 \leq x \leq 2\pi\} \quad xiv)$$

$$A = \{x \in \mathbb{R}: (x-1)(x^2-9)e^{3x+2} \leq 0\} \quad xv)$$

$$A = \{x \in \mathbb{R}: 0 \leq x \leq 2\pi, \sin x - \cos x \geq 0\} \quad xvi)$$

162) Determinate gli insiemi

$$A = \{x \in \mathbb{R}: e^{|x-1|} < e^x\} \quad B = \{x \in \mathbb{R}: |x^2 - |x+3|| < 3\}$$

$$A \cap [-7, 4]$$

$$B \setminus (-3, -1[)$$

163) Per ciascuno dei casi seguenti determinate gli insiemi $A, B, A \cup B, A \cap B, A \setminus B, B \setminus A$.

$$i) A = \{x \in \mathbb{R}: \sqrt{x^2 - 3x} \leq x - 1\} \quad B = \{x \in \mathbb{R}: x^2 - 2x - 8 \leq 0\}$$

$$ii) A = \{x \in \mathbb{R}: e^{x^2 - 6x + 2} > e^{2x - 5}\} \quad B = \{x \in \mathbb{R}: x^3 - 2x^2 - 5x + 6 \geq 0\}$$

$$iii) A = \{x \in \mathbb{R}: \frac{x+1}{x^2-4} < \frac{1}{x}\} \quad B = \{x \in \mathbb{R}: \log(x+4) > 0\}$$

164) Determinate i seguenti insiemi

$$i) A = \left\{ x \in \mathbb{R} : \left| \frac{2x+3}{3x+1} \right| = 1 \right\} \quad A \cup [1, +\infty[$$

$$ii) A = \left\{ x \in \mathbb{R} : \left| \frac{x-3}{x+2} \right| \leq 1 \right\} \quad A \setminus [3, +\infty[$$

$$iii) A = \left\{ x \in \mathbb{R} : ||x-2|-3| \leq 1 \right\} \quad iv) A = \left\{ x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \right. \\ \left. B = \left\{ x \in \mathbb{R} : \frac{2-x}{\sqrt{x^2-2}} < 1 \right\} \quad |\cos x - 1| \leq \cos x \right\}$$

$$A \cap B, A \setminus B, B \setminus A.$$

165) Disegnate il grafico delle seguenti funzioni specificando $\text{Im}f$, il $\text{min}f$ e $\text{max}f$ se esistono con i rispettivi punti di minimo e massimo, gli intervalli dove f è crescente e dove è decrescente.

$$i) f:]-\infty, 2] \rightarrow \mathbb{R} \quad f(x) = \begin{cases} e^x - 1 & x \leq 0 \\ -2x + 3 & 0 < x \leq 2 \end{cases} \quad \text{Eq.}^{\text{ue}} f(x) = \frac{1}{e} - 1$$

$$ii) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \log|x| & |x| \geq 1 \\ 1-x^2 & |x| < 1 \end{cases} \quad \text{Dis.}^{\text{ue}} f(x) > 2$$

$$iii) f(x) = |x-1| - |x+2| \quad f: \mathbb{R} \rightarrow \mathbb{R}$$

$$iv) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} x+3 & x \leq -1 \\ 2 & -1 < x < 2 \\ -x+4 & x \geq 2 \end{cases}$$

$$v) f: [-1, 4] \rightarrow \mathbb{R} \quad f(x) = |x|(1-|x|) \quad \text{Eq.}^{\text{ue}} f(x) = -2$$

$$vi) f:]-3, 5] \rightarrow \mathbb{R} \quad f(x) = |x|(1+|x|) \quad \text{Dis.}^{\text{ue}} f(x) \geq 6$$

$$vii) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} x & x \leq 0 \\ x^2 + 1 & 0 < x \leq 2 \\ e^{x-2} + 5 & x > 2 \end{cases} \quad \text{Eq.}^{\text{ue}} f(x) = 8$$

$$viii) f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \frac{1}{x} & x < 0 \\ (\log x) + 2 & x > 0 \end{cases} \quad \text{Dis.}^{\text{ue}} f(x) \leq -1$$

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$$ix) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} x+2 & x < -1 \\ | -x^2 + 1 | & x \geq -1 \end{cases} \quad \text{Eq.}^u f(x) = \frac{3}{4}$$

$$x) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} x^2 - 2x + 3 & x > 0 \\ 2 & x = 0 \\ \text{sen } x - 1 & x < 0 \end{cases} \quad \text{Eq.}^u f(x) = -1 - \frac{\sqrt{2}}{2}$$

$$xi) f:]0, 2] \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \sqrt{x-1} & 1 \leq x \leq 2 \\ (x-1)^2 + \frac{1}{2} & 0 < x < 1 \end{cases}$$

$$xii) f:]0, 2] \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \sqrt{x-1} & 1 \leq x \leq 2 \\ (x-1)^2 + 3 & 0 < x < 1 \end{cases}$$

$$xiii) f: [-2, 2] \rightarrow \mathbb{R} \quad f(x) = \begin{cases} 2x+2 & -2 \leq x \leq -1 \\ x^3 - 1 & -1 < x \leq 2 \end{cases} \quad \text{Eq.}^u f(x) = -\frac{7}{8}$$

$$xiv) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |x^2 + x| - x^2 + |x|$$

$$xv) f: [-1, 4] \rightarrow \mathbb{R} \quad f(x) = \max(|x+1|, 2)$$

$$xvi) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} e^{x-1} & x < 0 \\ -\frac{2}{3}x + 2 & 0 \leq x \leq 3 \\ \log(x-2) & x > 3 \end{cases} \quad \text{Dis.}^u f(x) \geq 2$$

$$xvii) f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \quad f(x) = \log|x-1| \quad \text{Dis.}^u f(x) < -2$$

$$xviii) f: [-1, 3] \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \sqrt{|x|} & -1 \leq x \leq 0 \\ 2x^2 & 0 < x < 1 \\ 1 & 1 \leq x < 2 \text{ o } 2 < x \leq 3 \\ 2 & x = 2 \end{cases}$$

$$xix) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \frac{2x^2 - 4x}{|x+1| + |x-3| - 2}$$

$$xx) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} ||x+2|-1| & x < 0 \\ e^{-x} - 1 & x \geq 0 \end{cases}$$

$$xxi) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} 2^x & x \leq 0 \\ \log(x+1) & x > 0 \end{cases} \quad \text{Dis.}^u f(x) < 1$$

$$xxii) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} -x-1 & x < 0 \\ -\sqrt{1-x^2} & 0 \leq x < 1 \\ \log x & x \geq 1 \end{cases}$$

$$xxiii) f: \mathbb{R} \setminus \{3\} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} -x-3 & x < -2 \\ \sqrt{4-x^2} & -2 \leq x \leq 2 \\ x-2 & 2 < x < 3 \\ -\sqrt{x-3} & x > 3 \end{cases}$$

$$xxiv) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} -x^2 - 2x & x < -1 \\ |x| & -1 \leq x \leq 1 \\ x^2 - 2x + 3 & x > 1 \end{cases}$$

$$xxv) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \begin{cases} \frac{1}{x} & x < 0 \\ \sin x & 0 \leq x \leq 4\pi \\ -x + 4\pi & x > 4\pi \end{cases} \quad \text{Dis.}^{\text{ue}} -1 \leq f(x) < 0$$

166) Determinate il dominio delle seguenti funzioni:

$$i) f(x) = \sqrt{x^4 - 2x^2 - 3} \quad ii) f(x) = \log(\sin x - \cos x)$$

$$iii) f(x) = \log(3x^4 - x^3 - 3x + 1) \quad iv) \sqrt{1 + \frac{1}{x}} = f(x)$$

$$v) f(x) = \frac{3x-2}{x-x^2} \quad vi) f(x) = \sqrt{\log(1+x^2)} \quad vii) f(x) = \frac{1}{|x-2|}$$

20) Le due funzioni $f(x) = \frac{\sqrt{x}}{\sqrt{x-1}}$ e $g(x) = \sqrt{\frac{x}{x-1}}$ sono uguali?
(giustificare la risposta)

167) Determinate il dominio delle seguenti funzioni:

$$a) f(x) = \log(x^2 - 5x + 6) + \sqrt{4x+3}$$

$$b) f(x) = e^{x-3} + \sqrt{x^2 - 1}$$

$$c) f(x) = \sin\left(\frac{x}{3}\right) - \sqrt{9-x^2} + \frac{\log(1-2x)}{e^{2x}}$$

$$d) f(x) = \frac{\cos x}{9x - 4x^2 - 2} + \frac{1}{\sqrt{x}} \quad i) f(x) = \frac{\sqrt{6-x^2+x}}{2-3x}$$

$$e) f(x) = \frac{\log(1 - \frac{3}{2}x - x^2)}{16x^2 - 1} + \sqrt{5x+7}$$

$$f) f(x) = \frac{\log((x+2)(x+1)-2)}{x^2 + 10x + 25}$$

168) Disegnate il grafico delle seguenti funzioni specificando ^{il dominio,} l'immagine e il numero delle soluzioni dell'eq.^{ne} $f(x)=k$ al variare di $k \in \mathbb{R}$.

$$a) f(x) = \begin{cases} \frac{1}{|x|} & x \leq -1 \\ \sqrt{4-x} & -1 < x \leq 4 \\ e^{x-4} & x > 4 \end{cases} \quad \begin{array}{l} \text{intervalli su cui } f \text{ è crescente o} \\ \text{decrescente} \\ \text{DIS.}^{\text{ne}} f(x) \geq 1 \end{array}$$

$$b) f(x) = \begin{cases} \log(-x) & x < 0 \\ -e^x & x \geq 0 \end{cases} \quad \begin{array}{l} \text{intervalli su cui } f \text{ è crescente o} \\ \text{decrescente} \end{array}$$

$$c) f(x) = \begin{cases} 1 + e^{-x-1} & x \leq 0 \\ \frac{1}{(x-2)^2} & x > 2 \\ \log|x-2| & 0 < x < 2 \end{cases} \quad \begin{array}{l} \text{Eq.}^{\text{ne}} f(x) = \frac{1}{2} \\ \text{intervalli su cui } f \text{ è crescente} \\ \text{o decrescente} \end{array}$$

$$d) f(x) = |2-3x| \text{ su } [-3, 2[$$

$$e) f(x) = ||\log x| - 1| \quad \text{Eq.}^{\text{ne}} f(x) = 1$$

$$f) f(x) = \begin{cases} \sin|x| & -2\pi \leq x \leq 0 \\ |x^2-3x| & 0 < x \leq 4 \end{cases} \quad \begin{array}{l} \text{P.T. di Massimo} \\ \text{e minimo locale} \end{array}$$

$$g) f(x) = |-2 + \sqrt{x}|$$

$$h) f(x) = |x^2 - x - 2|$$

$$i) f(x) = -|\log(x+1) - 1|$$

$$k) f(x) = \begin{cases} e^{1-x} + 1 & x \geq 1 \\ e^{x+1} + 2 & x < 1 \end{cases} \quad \begin{array}{l} \text{se è iniettiva} \\ \text{DIS.}^{\text{ne}} \frac{3}{2} < f(x) < 3 \end{array}$$

169) Ancora DOMINI :

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$$a) f(x) = \sqrt{\log(1-2x)-1}$$

$$b) f(x) = \frac{1}{\log_3 x - 2}$$

$$c) f(x) = \sqrt{\log_{\frac{1}{2}}(x-1)}$$

$$d) f(x) = \frac{1}{\sqrt{1-\log x}}$$

$$e) f(x) = \sqrt{\frac{|x-2|-3}{|x|+2x-1}}$$