

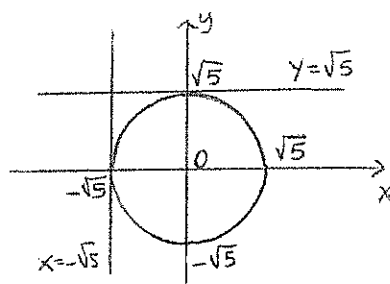
SOLUZIONI GEOMETRIA ANALITICA

El. Mat.
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170) i) $x^2 + y^2 = 5$ $C(0,0)$ $R = \sqrt{5}$

ii) $y = \sqrt{5}$ iii) $x = -\sqrt{5}$

iv) $C(0,0)$ $R = 7$ $x^2 + y^2 = 49$



171) i) $(x-1)^2 + (y-3)^2 = 16$

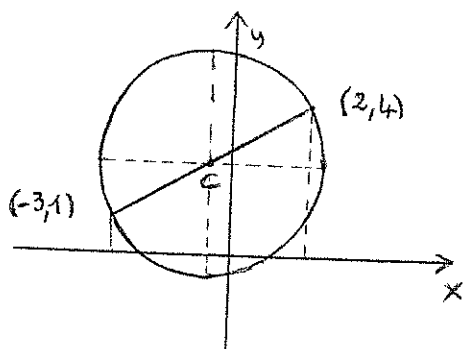
ii) $(x - \frac{3}{2})^2 + (y + \frac{1}{2})^2 = R^2$ per $A = (1, \frac{3}{2})$ $\frac{1}{4} + 4 = R^2$ $R^2 = \frac{17}{4}$ ($R = \frac{\sqrt{17}}{2}$)

$(x - \frac{3}{2})^2 + (y + \frac{1}{2})^2 = \frac{17}{4}$ (oppure $R = \text{dist}(C, A) = \sqrt{(\frac{1}{2})^2 + (-2)^2} = \sqrt{\frac{17}{4}}$)

iii) è la circonf. di centro $C =$ punto medio del segmento AB

e $R = \frac{1}{2} \text{dist}(A, B)$

$C = (-\frac{3+2}{2}, \frac{1+4}{2}) = (-\frac{1}{2}, \frac{5}{2})$ $R = \frac{1}{2} \sqrt{5^2 + 3^2} = \frac{\sqrt{34}}{2}$



eq. $(x + \frac{1}{2})^2 + (y - \frac{5}{2})^2 = \frac{34}{4} = \frac{17}{2}$

172) i) $(x-1)^2 + (y+2)^2 = 3$

ii) $R = \text{dist}(C, (0,0)) = \sqrt{1 + \frac{25}{49}} = \frac{\sqrt{74}}{7}$

iii) $C \begin{cases} y = -2x \\ y = -\frac{3}{4}x - \frac{1}{4} \end{cases} \begin{cases} -2x = -\frac{3}{4}x - \frac{1}{4} \\ \dots \end{cases} \begin{cases} \frac{5}{4}x = -\frac{1}{4} \\ \dots \end{cases} \begin{cases} x = -\frac{1}{5} \\ y = -\frac{2}{5} \end{cases}$

$C(\frac{1}{5}, -\frac{2}{5})$ $R = \sqrt{2}$ $(x - \frac{1}{5})^2 + (y + \frac{2}{5})^2 = 2$

173) i) $C(0,0)$ $R = 4$ ii) $C(-1, 3)$ $R = 3$ iii) $C(0,0)$ $R = 2\sqrt{2}$

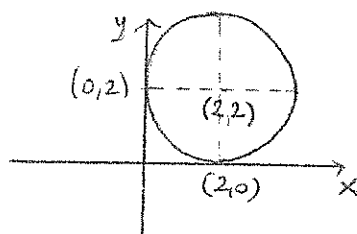
iv) $(x-1)^2 + y^2 = 8+1=9$ $C(1,0)$ $R = 3$ v) $x^2 + (y+2)^2 = 5$ $C(0, -2)$ $R = \sqrt{5}$

vi) $x^2 + y^2 + \frac{2}{3}x + 10y - \frac{122}{3} = 0$ $(x + \frac{1}{3})^2 + (y+5)^2 = \frac{122}{3} + \frac{1}{9} + 25 = \frac{592}{9}$

$C(-\frac{1}{3}, -5)$ e $R = \frac{\sqrt{592}}{3} = \frac{\sqrt{2^4 \cdot 37}}{3} = \frac{4}{3} \sqrt{37} \approx 8,11$

174) l'eq. $(x-1)^2 + (y+2)^2 = -1$ non ci sono p.ti che verificano l'eq. quindi NON rappresenta una circonferenza.

175) $C(2,2)$ $R=2$ è tangente ad entrambi gli assi



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176) i) $y = a(x-x_v)^2 + y_v$ $V(0,0)$

$y = ax^2$

A) $\begin{cases} y = \frac{3}{2}x + 1 \\ y = 2x - 3 \end{cases} \Rightarrow \begin{cases} 2x - 3 = \frac{3}{2}x + 1 \\ \dots \end{cases} \Rightarrow \begin{cases} \frac{1}{2}x = 4 \\ \dots \end{cases} \Rightarrow \begin{cases} x = 8 \\ y = 13 \end{cases} \Rightarrow A(8,13)$

$y = ax^2$ per A $13 = a \cdot 64 \Rightarrow a = \frac{13}{64}$ $y = \frac{13}{64}x^2$ Eq.

ii) $y = ax^2 + bx + c$ $\begin{cases} \text{per } (0,0) & c=0 \\ \text{per } (2,3) & 3 = 4a + 2b \\ \text{per } (-1,4) & 4 = a - b \end{cases} \Rightarrow \begin{cases} a = \frac{11}{6} \\ b = -\frac{13}{6} \end{cases}$

Eq. $y = \frac{11}{6}x^2 - \frac{13}{6}x$

iii) $y = a(x-x_v)^2 + y_v$ $V(2,-3)$ $y = a(x-2)^2 - 3$ per $(0,0)$

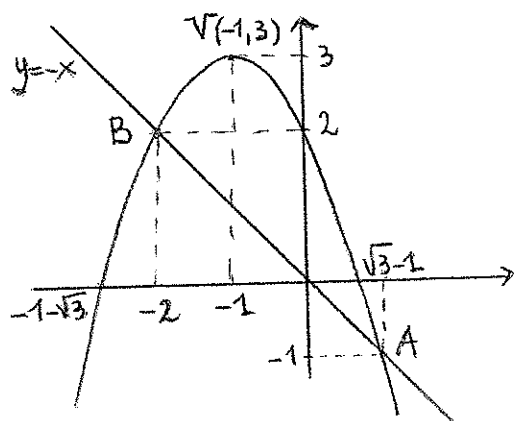
$0 = 4a - 3 \Rightarrow a = \frac{3}{4}$ Eq. $y = \frac{3}{4}(x-2)^2 - 3 = \frac{3}{4}x^2 - 3x$

177) $\begin{cases} y = 2x \\ y = 2x^2 \end{cases} \Rightarrow \begin{cases} 2x^2 = 2x \\ \dots \end{cases} \Rightarrow \begin{cases} 2x(x-1) = 0 \\ y = 2x \end{cases} \Rightarrow \begin{cases} x = 0 \text{ o } x = 1 \\ y = 2x \end{cases}$

INTERSEZIONI A(0,0) B(1,2)

178) i) $\begin{cases} y = -x \\ y = -x^2 - 2x + 2 \end{cases} \Rightarrow \begin{cases} x^2 + x - 2 = 0 \\ y = -x \end{cases} \Rightarrow \begin{cases} x = 1 \text{ o } x = -2 \\ y = -x \end{cases}$

A=(1,-1) B=(-2,2)



ii) $\begin{cases} y = 3 - x^2 \\ y = x^2 - 2x - 1 \end{cases}$

$2x^2 - 2x - 4 = 0$

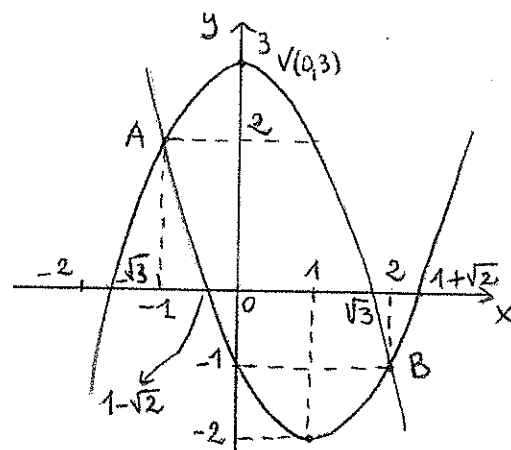
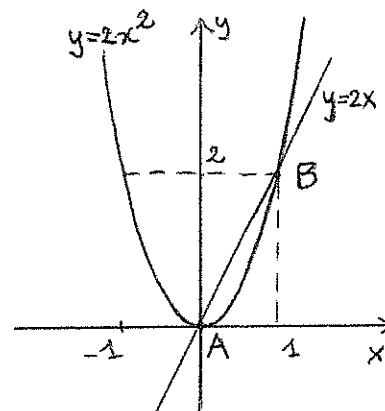
$y = 3 - x^2$

$x^2 - x - 2 = 0$

$y = 3 - x^2$

$x = -1 \text{ o } x = 2$

$y = 3 - x^2$ A(-1,2) B(2,-1)



179) i) $C(0,0)$ semiaxi (3,1)

ii) $C(0,0)$ semiaxi (1,4)

iii) $x^2 + \frac{y^2}{4} = 1$ $C(0,0)$ sem (1,2)

iv) $x^2 + \frac{y^2}{\frac{1}{9}} = 1$ $C(0,0)$ sem (1, $\frac{1}{3}$)

v) $C(1,-2)$ sem (2,4)

vi) $\frac{x^2}{\frac{1}{4}} + \frac{(y-3)^2}{4} = 1$ $C(0,3)$ sem (1,2)

vii) $9(x+1)^2 + y^2 = 9$ $(x+1)^2 + \frac{y^2}{9} = 1$
 $C(-1,0)$ sem (1,3)

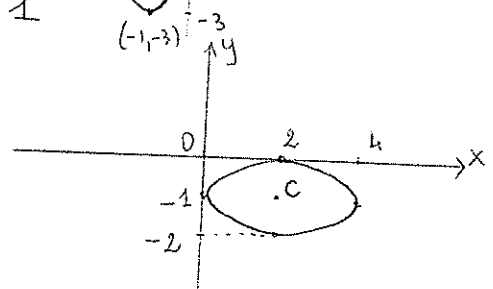
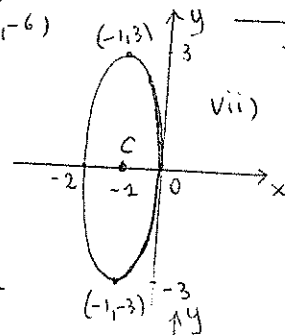
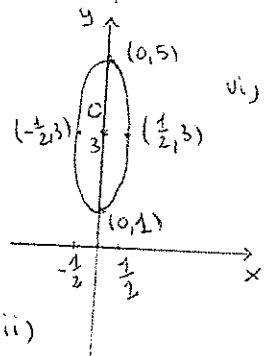
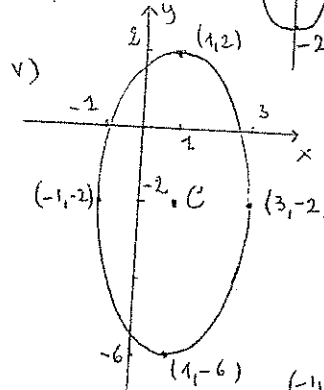
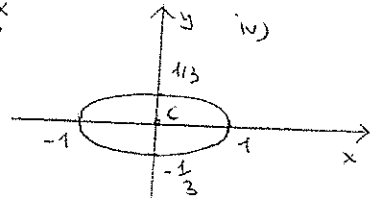
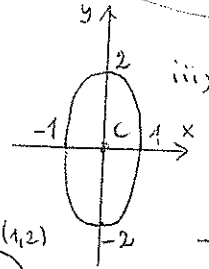
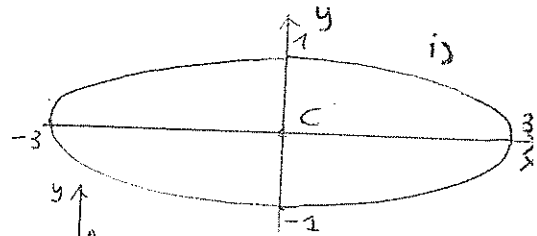
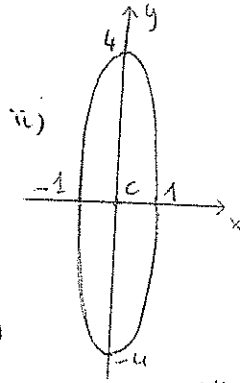
viii) $(x-2)^2 + 4(y+1)^2 = 4$ $\frac{(x-2)^2}{4} + (y+1)^2 = 1$
 $C(2,-1)$ sem (2,1)

180) i) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ ii) $\frac{(x-1)^2}{9} + (y+1)^2 = 1$

iii) $\frac{(x-2)^2}{16} + \frac{y^2}{4} = 1$ iv) $x^2 + \frac{y^2}{16} = 1$

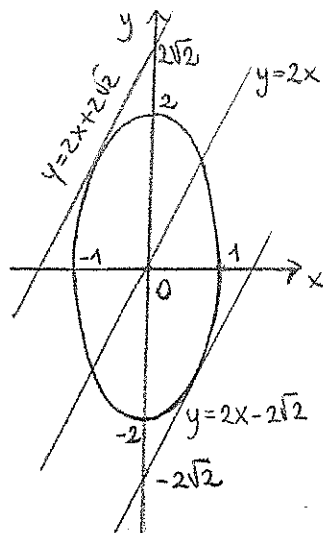
181) ha $C(0,0)$ semiaxi (5,4) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

182) $\begin{cases} y = 2x + q \\ x^2 + \frac{y^2}{4} = 1 \end{cases} \dots \begin{cases} x^2 + \frac{1}{4}(2x+q)^2 = 1 \\ 2x^2 + xq + \frac{1}{4}q^2 - 1 = 0 \end{cases} \dots$
 $\Delta = q^2 - 4 \cdot 2 \left(\frac{1}{4}q^2 - 1 \right) = q^2 - 2q^2 + 8 = 8 - q^2$



ci sono intersezioni se $8 - q^2 \geq 0$
 cioè $q^2 \leq 8 \Leftrightarrow -2\sqrt{2} \leq q \leq 2\sqrt{2}$

ELLISSE $(0,0)$ $a=1$ $b=2$ $y=2x+q$
 retta $m=2$



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183) la retta è $x = \frac{x}{2}$ che è una retta verticale, quindi il coefficiente angolare non è definito.

184) i) riferita alle bisettrici, per $(1,0)$ e $(-1,0)$

ii) riferita agli assi, per $(1,1)$ e $(-1,-1)$

iii) riferita alle bisettrici per $(0,2)$ e $(0,-2)$

$$\frac{y^2}{4} - \frac{x^2}{4} = 1 \quad a^2 = 4 = b^2$$

l'eq. si scrive
 anche $y^2 - x^2 = b^2$

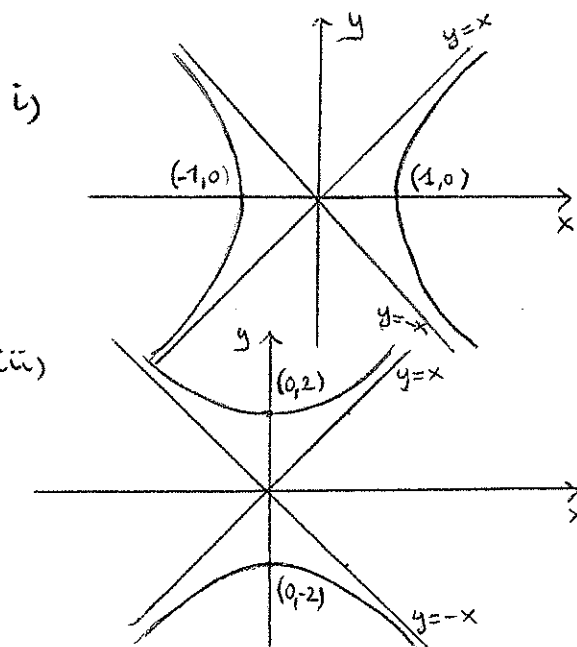
iv) riferita agli assi per $(1,-2)$ e $(-1,2)$

v) riferita a $y = \pm \frac{3}{2}x$ per $(2,0)$ e $(-2,0)$

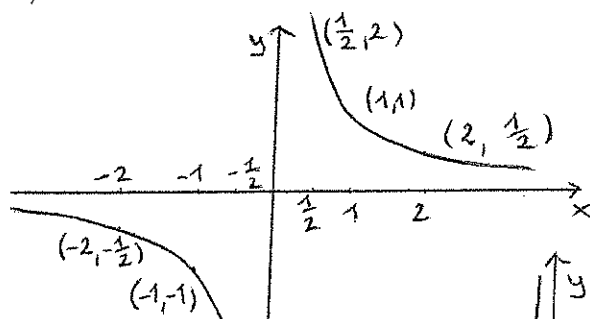
vi) riferita a $y = \pm 5x$ per $(0,5)$ e $(0,-5)$

vii) riferita agli assi per $(2,2)$ e $(-2,-2)$

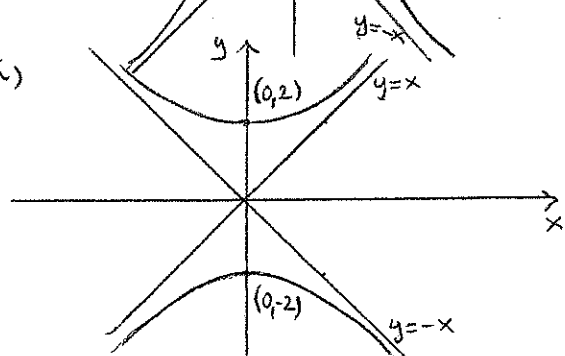
viii) riferita agli assi per $(1,-3)$ e $(-1,3)$ (anche $(3,-1)$ e $(-3,1)$)



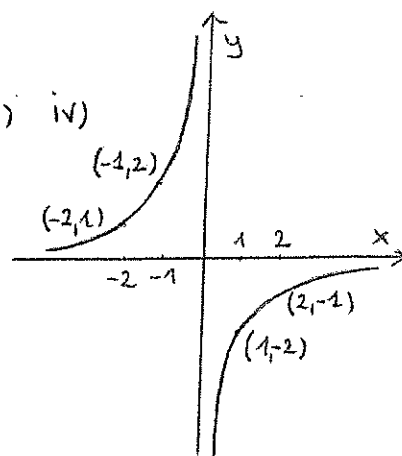
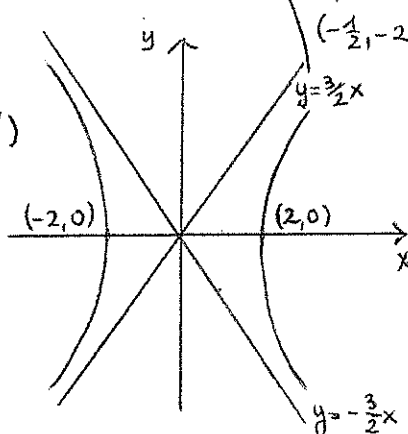
ii)



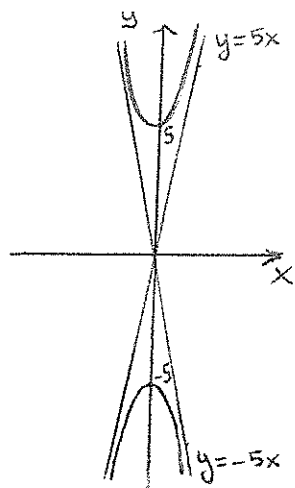
iii)



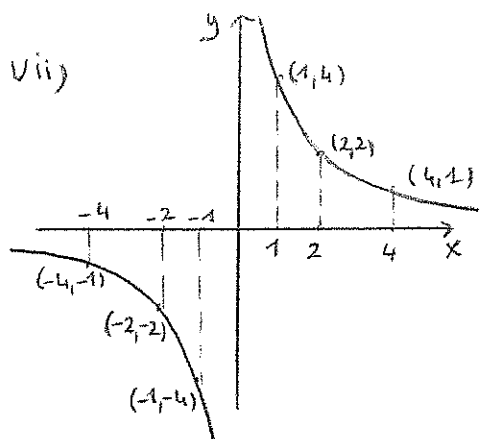
iv)



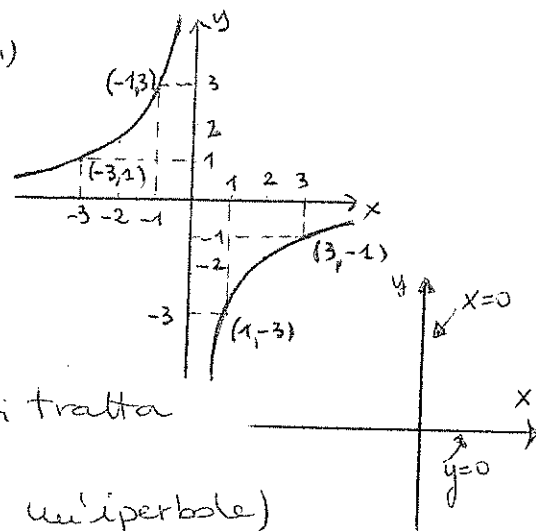
vi)



vii)



viii)

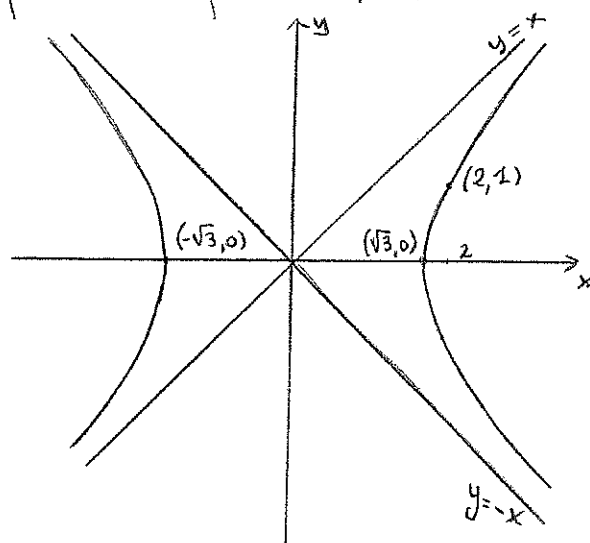


185) $xy=0 \Leftrightarrow x=0 \cup y=0$ quindi si tratta dei due assi cartesiani (e non è un'iperbole)

186) L'iperbole equilatera non è riferita agli assi, bensì alle bisettrici $y=\pm x$. Dovendo passare per $(2,1)$ dev'essere del tipo $x^2 - y^2 = a^2$, da cui per $x=2$ e $y=1$ si ottiene $4-1=a^2$, $a^2=3$

$$x^2 - y^2 = 3$$

passa per $(\sqrt{3}, 0)$ e $(-\sqrt{3}, 0)$

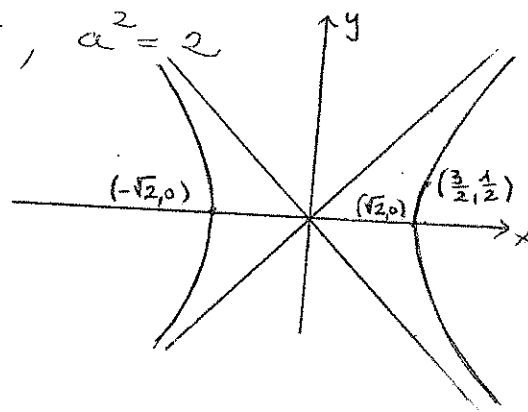


Anche l'iperbole equilatera per $(\frac{3}{2}, \frac{1}{2})$ ha la forma

$$x^2 - y^2 = a^2, \text{ da cui } \frac{9}{4} - \frac{1}{4} = a^2, a^2 = 2$$

$$x^2 - y^2 = 2$$

(passa per $(\sqrt{2}, 0)$ e $(-\sqrt{2}, 0)$)



$$187) \left\{ \begin{array}{l} y = \frac{1}{2}x + \frac{1}{2} \\ y = -\frac{3}{2}x - 1 \end{array} \right\} \dots \left\{ \begin{array}{l} 2x = -\frac{3}{2} \\ \dots \end{array} \right\} \left\{ \begin{array}{l} x = -\frac{3}{4} \\ y = \frac{1}{8} \end{array} \right.$$

$$A(-\frac{3}{4}, \frac{1}{8})$$

$$x^2 - y^2 = a^2 \quad \frac{9}{16} - \frac{1}{64} = \frac{36-1}{64} = \frac{35}{64}$$

$$x^2 - y^2 = \frac{35}{64} \quad (\text{para per } (\pm \frac{\sqrt{35}}{8}, 0))$$

$$188) \text{ asintoti } y = \pm \frac{5}{2}x \quad b=5 \quad a=2 \quad \frac{x^2}{4} - \frac{y^2}{25} = 1$$

$$189) \text{ asintoti } y = \pm \frac{4}{3}x \quad b=4 \quad a=3 \quad \frac{y^2}{16} - \frac{x^2}{9} = 1$$