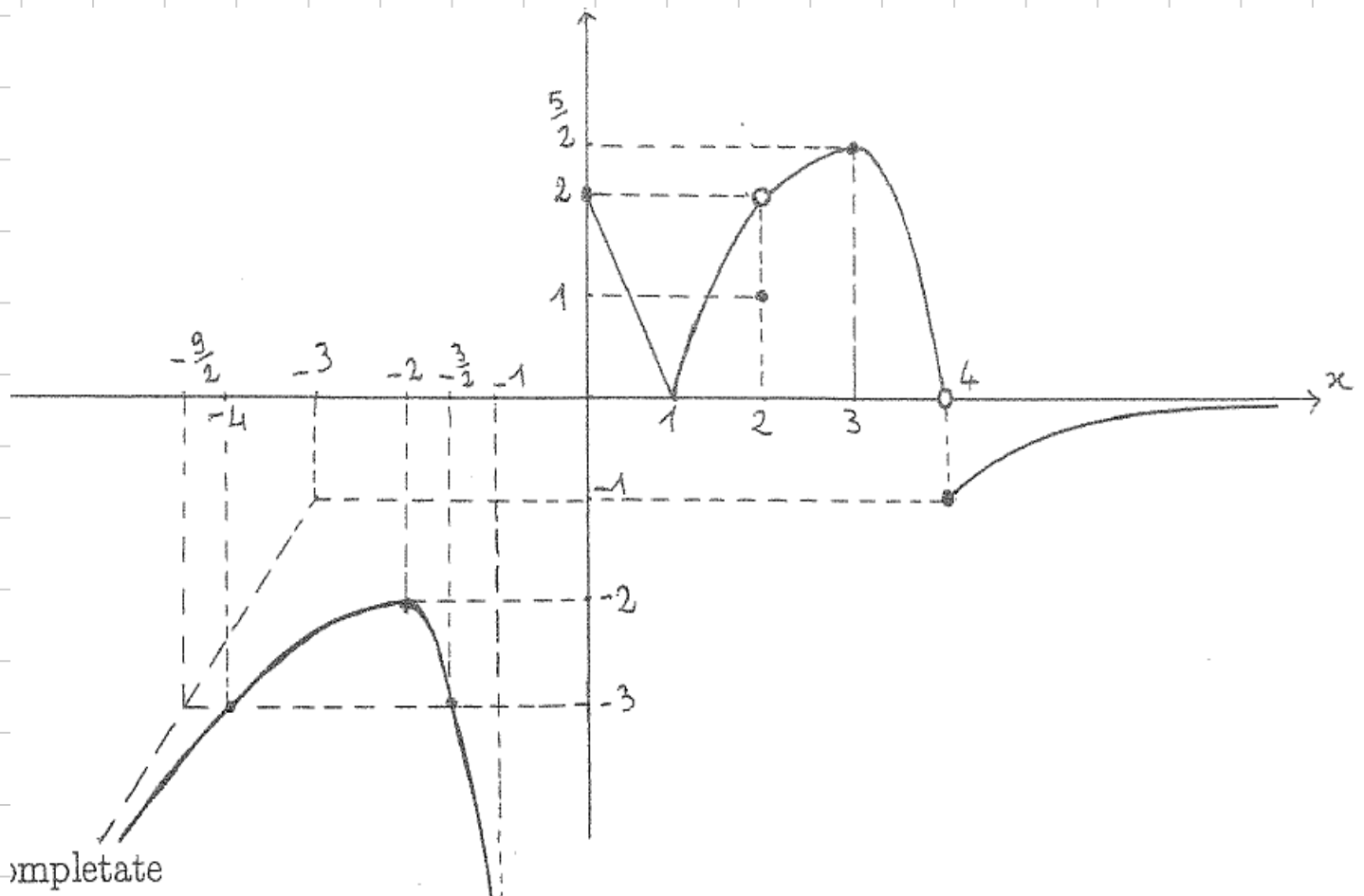


Ex 1



$$\text{Dom} = (-\infty, -1) \cup [0, +\infty)$$

$$\text{Imm} = (-\infty, -2] \cup [-1, \frac{5}{2}]$$

FUNZIONE NON E' INT.

$$f(-4) = f(-\frac{3}{2}) = -3$$

FUNZIONE NON E' SURJ.

$$f(-\frac{1}{2}) = \text{NON DEFINITA} \quad f(2) = 1$$

$$f^{-1}(-3) = \left\{-4, -\frac{3}{2}\right\} \quad f^{-1}\left(-\frac{3}{2}\right) = \emptyset$$

LA FUNZIONE NON HA MINIMO

LA FUNZIONE HA MASSIMO NEL PUNTO
 $\left(3, \frac{5}{2}\right)$

MINIMI LOCALI: $(1, 0); (2, 1); (4, -1)$

MASSIMI LOCALI: $(-2, -2); (0, 2); \left(3, \frac{5}{2}\right)$

MONOTONIA

CRES. IN $(-\infty, -2] \cup \left[\frac{1}{2}, 2\right) \cup (2, 3] \cup [4, +\infty)$

DECRESC. IN $[-2, -1) \cup [0, 1] \cup [3, 4]$

$\overline{k \in \mathbb{R}}$

$$\left\{ f(x) = k \right\}$$

$-\infty < k < -2$ 2 SOLUZIONI.

$k = -2$ 1 SOLUZIONE

$-2 < k < -1$ 0 SOLUZIONI

$-1 \leq k \leq 0$ 1 SOLUZIONE

$$0 < k < 1$$

3 SOLUZIONI

$$k = 1$$

4 SOLUZIONI

$$1 < k < 2$$

3 SOLUZIONI

$$(k = 2$$

2 SOLUZIONI)

$$2 \leq k < \frac{5}{2}$$

2 SOLUZIONI

$$k = \frac{5}{2}$$

1 SOLUZIONE

$$k > \frac{5}{2}$$

0 SOLUZIONI

$x \in \text{Dom } f$

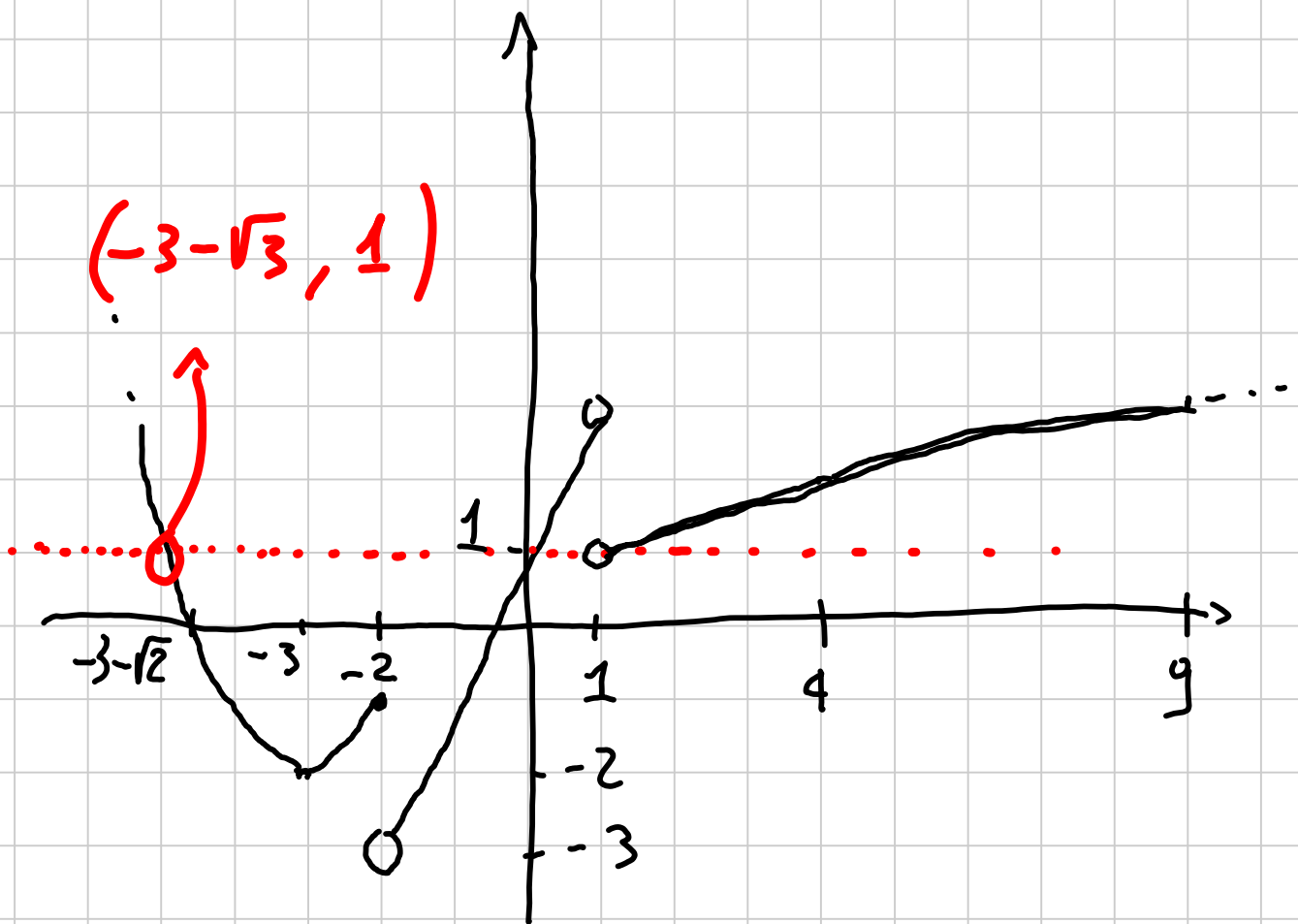
$$\left\{ f(x) < \frac{5}{2} \right\} = \text{Dom } f \setminus \{3\}$$

Ex 2

$$f(x) = \begin{cases} x^2 + 6x + 7 & x \leq -2 \\ 2x + 1 & -2 < x < 1 \\ \sqrt{x} & x > 1 \end{cases}$$

DISEGNARE IL GRAFICO DI $f(x)$

$$\text{Dom } f = \mathbb{R} \setminus \{1\}$$



$$\boxed{x \leq -2}$$

$$f(x) = x^2 + 6x + 7$$

$$x_v = \frac{-b}{2a} = \frac{-6}{2} = -3$$

$$y_v = (-3)^2 + 6(-3) + 7 = -2$$

$$f(-2) = -1$$

$$f(x) = 0 \Leftrightarrow x^2 + 6x + 7 = 0$$

$$\Delta = 36 - 28 = 8 > 0$$

$$x_{1,2} = \frac{-6 \pm 2\sqrt{2}}{2} = -3 \pm \sqrt{2}$$

$$x_1 = -3 - \sqrt{2} < -2$$

$$x_2 = -3 + \sqrt{2} > -2$$

$$\boxed{-2 < x < 1}$$

$$f(x) = 2x + 1$$

$$f(-2) = -3$$

$$f(1) = 3$$

$$f(0) = 1$$

$$\boxed{x > 1}$$

$$f(x) = \sqrt{x}$$

$$f(1) = 1$$

$$f(4) = 2$$

$$f(9) = 3$$

$$I_m f = (-3, +\infty)$$

NO MIN / MAX GLOBALE

MIN LOCALE $(-3, -2)$

MAX LOCALE $(-2, -1)$

$$f(x) > 1$$

VERO SE $\boxed{x > 1}$ ($\sqrt{x} > 1$)

$$f(x) = 2x + 1 > 1 \quad (-2 < x < 1)$$

$$\Leftrightarrow 2x > 0 \Leftrightarrow \boxed{\begin{cases} x > 0 \\ x < 1 \end{cases}}$$

$$x \leq -2$$

$$\begin{cases} x^2 + 6x + 7 > 1 \\ x \leq -2 \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} x^2 + 6x + 6 > 0 \\ x \leq -2 \end{cases}$$

$$\Delta = 36 - 24 = 12$$

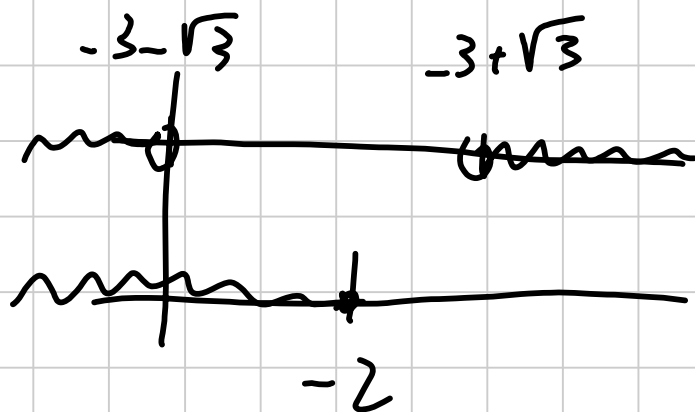
$$x^2 + 6x + 6 = 0 \Leftrightarrow$$

$$x_{1,2} = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$\Leftrightarrow \begin{cases} x < -3 - \sqrt{3} \vee x > -3 + \sqrt{3} \\ x \leq -2 \end{cases} \Leftrightarrow \begin{aligned} x_1 &= -3 - \sqrt{3} \\ x_2 &= -3 + \sqrt{3} \end{aligned}$$

$$-3 + \sqrt{3} > -2$$

$$\Leftrightarrow x < -3 - \sqrt{3}$$



TROVARE SOLUT. $0 \leq x < 2\pi$ d:

$$\bullet \cos^2(x) = 1 \Leftrightarrow \cos^2(x) - 1 = 0$$

$$\Leftrightarrow (\cos(x) - 1)(\cos(x) + 1) = 0$$

$$\Leftrightarrow \vee \cos(x) = 1 \Leftrightarrow x = 0$$

$$\cos(x) = -1 \Leftrightarrow x = \pi$$

$$\bullet (\tan(x))^2 = \frac{1}{3} \begin{cases} \nearrow \tan(x) = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3} \\ \searrow \tan(x) = -\sqrt{\frac{1}{3}} = -\frac{\sqrt{3}}{3} \end{cases}$$

$$\left[a^2 = b^2 \Leftrightarrow \vee \begin{matrix} a = b \\ a = -b \end{matrix} \right]$$

$$\tan(x) = \frac{\sqrt{3}}{3} \Rightarrow x = \frac{\pi}{6} \vee x = \frac{7}{6}\pi$$

$$\tan(x) = -\frac{\sqrt{3}}{3} \Leftrightarrow x = \frac{5}{6}\pi \vee x = \frac{11}{6}\pi$$

$$\left[\tan(x + \pi) = \tan(x) \right]$$

$$\sin(\pi + x) = -\sin(x)$$

$$\cos(\pi + x) = -\cos(x)$$

$$\sin(x) = \frac{1 + \sqrt{5}}{2}$$

$$\frac{1 + \sqrt{5}}{2} > \frac{1 + \sqrt{4}}{2} =$$

No solution in $\mathbb{R}$

$$= \frac{1 + 2}{2} = \frac{3}{2}$$

$$\bullet \cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2} \quad y = \frac{x}{2}$$

$$\cos(y) = \frac{\sqrt{3}}{2} \Leftrightarrow y = \frac{\pi}{6} \vee y = \frac{11}{6}\pi$$

$$\Leftrightarrow \frac{x}{2} = \frac{\pi}{6} \vee \frac{x}{2} = \frac{11}{6}\pi$$

$$\Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{11}{3}\pi$$

$$0 \leq x < 2\pi, \quad \boxed{x = \frac{\pi}{3} \text{ ok}} \quad \underline{x = \frac{11}{3}\pi \text{ NO}}$$

$$\bullet (\tan(x)^2 - 3) \cos(x) = 0 \quad \Leftrightarrow$$

$$(\tan(x))^2 - 3 = 0$$

$$\vee \cos(x) = 0$$

CE

$$\cos(x) \neq 0$$

$$\leftarrow \Leftrightarrow x \neq \frac{\pi}{2} \wedge x \neq \frac{3}{2}\pi$$

$$\Leftrightarrow \tan(x) = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{4}{3}\pi$$

$$\vee \tan(x) = -\sqrt{3} \Leftrightarrow x = \frac{2}{3}\pi \vee x = \frac{5}{3}\pi$$

$$\text{Sol.} = \left\{ \frac{\pi}{3}, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi \right\}$$