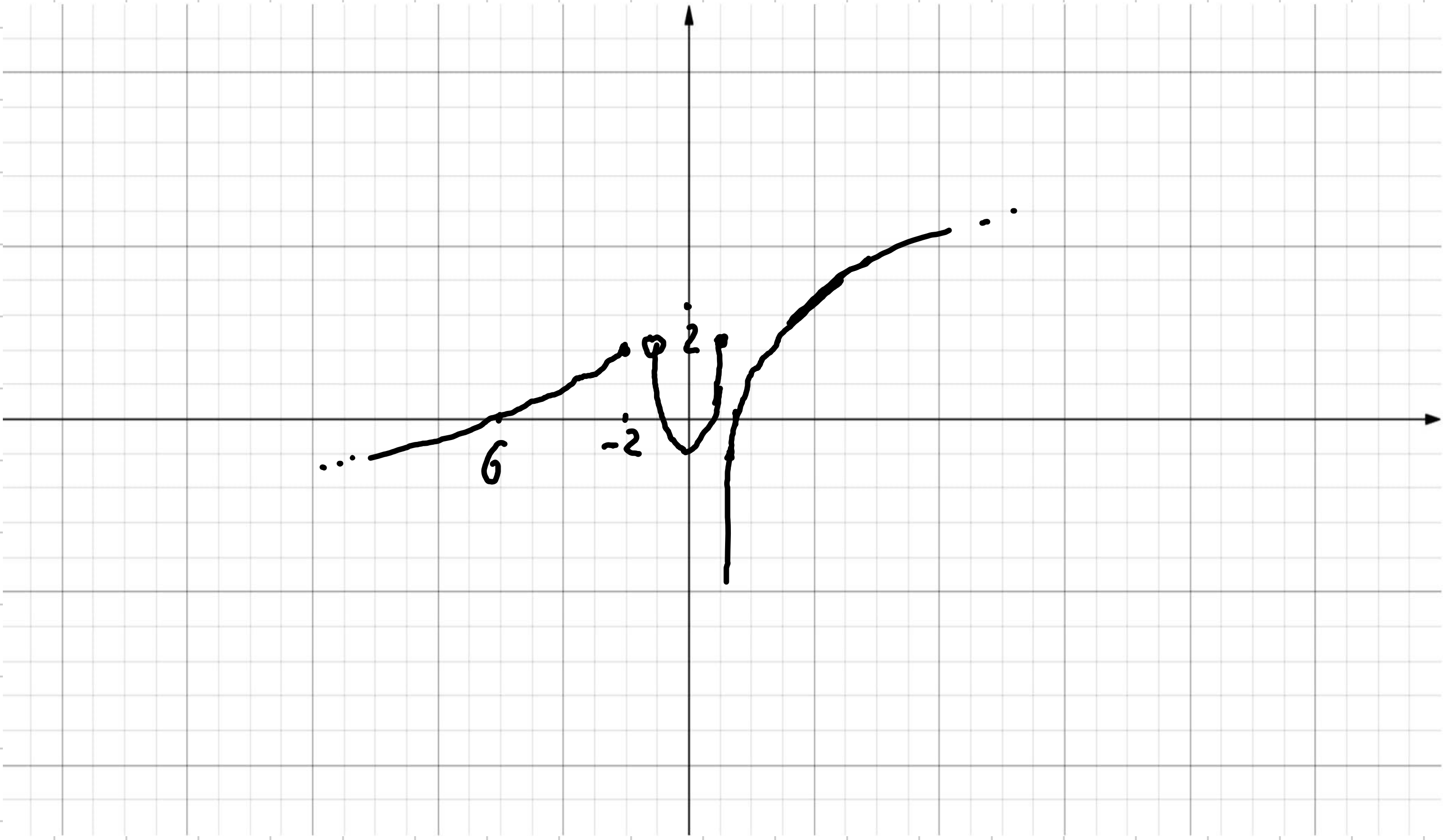
Ex 1



DISEGNARE IL GRAFICO DELLA FUNZIONE

$$f(x) = \begin{cases} -\sqrt{-x-2}+2 & x \leq -1 \\ 3|x^3|-1 & -1 < x \leq 1 \\ \log(x-1)+2 & x > 1 \end{cases}$$

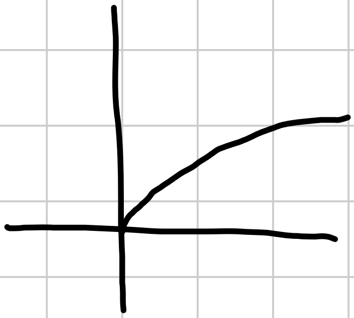
$$\mathrm{dom} f \quad \begin{cases} x \leq -1 \\ -x-2 \geq 0 \end{cases} \quad \begin{cases} x \leq -1 \\ x \leq -2 \end{cases} \Rightarrow x \leq -2$$

$\boxed{x \le -2}$

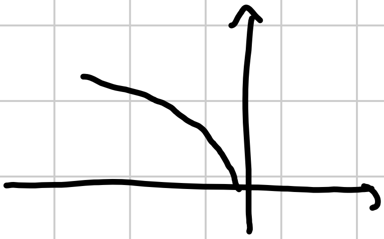
$$-\sqrt{-x-2}+2 = -\sqrt{-\underbrace{(x+2)}}+2$$

$$x=-2 \qquad f(-2)=2$$

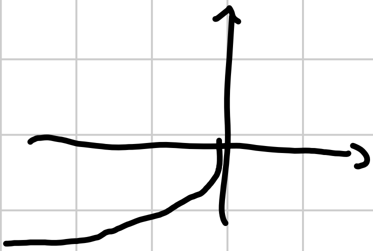
$\sqrt{x}$



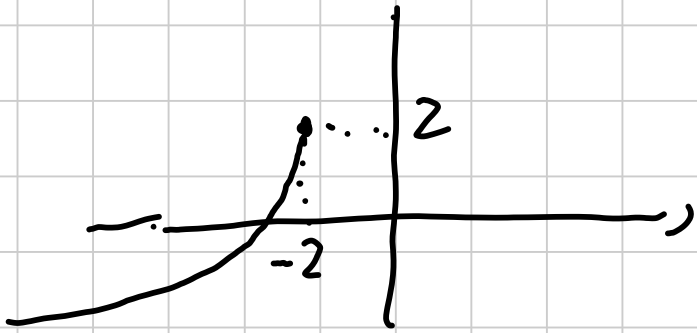
$\sqrt{-x}$



$-\sqrt{-x}$



$-\sqrt{-(x+2)}+2$



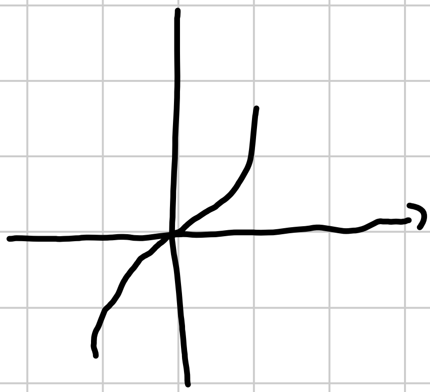
$$f(x)=0 \iff -\sqrt{-x-2}+2=0 \iff$$

$$\sqrt{-x-2}=2 \iff \begin{cases} -x-2 \ge 0 \\ 2 \ge 0 \\ -x-2=4 \end{cases}$$

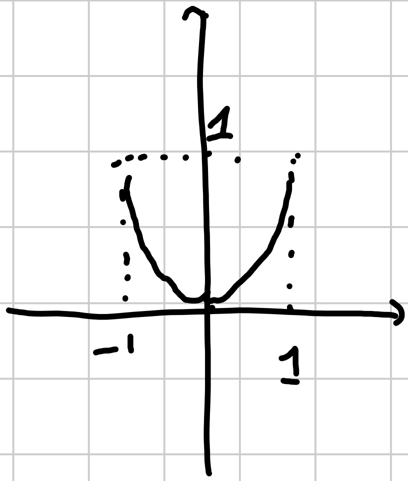
$$\begin{cases} x < -2 \\ \vee \\ x = -6 \end{cases}$$

$-1 < x \leq 1$ $\qquad f(x) = 3|x^3| - 1$ $\qquad$ HA SENSO IN $(-1, 1]$

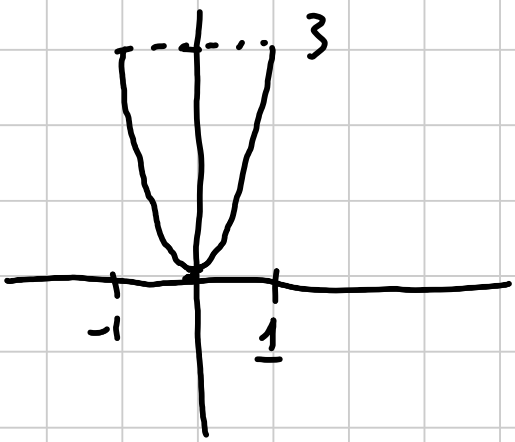
x^3



$|x^3| = |x|^3$



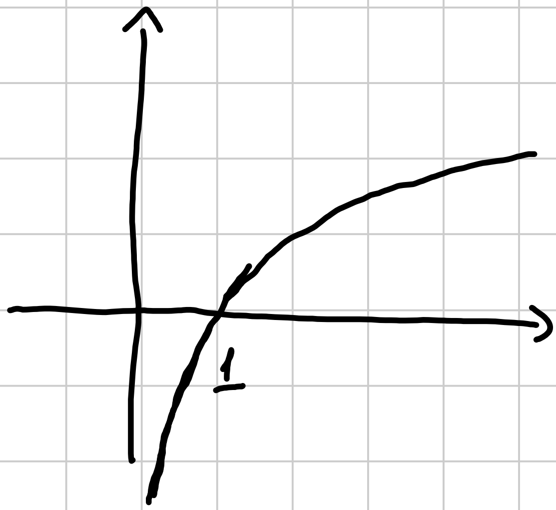
$3|x^3|$



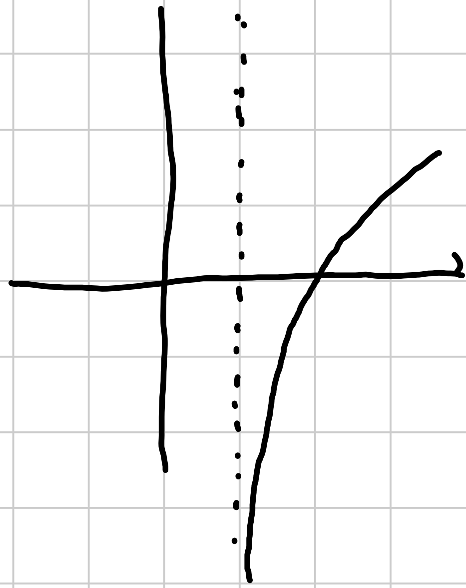
$\boxed{x > 1}$

$$\begin{cases} x > 1 \\ x - 1 > 0 \end{cases} \Rightarrow \boxed{x > 1}$$

$\log x$



$\log(x-1)$



$$\log(x-1)+2=0 \iff$$

$$\log(x-1)=-2$$

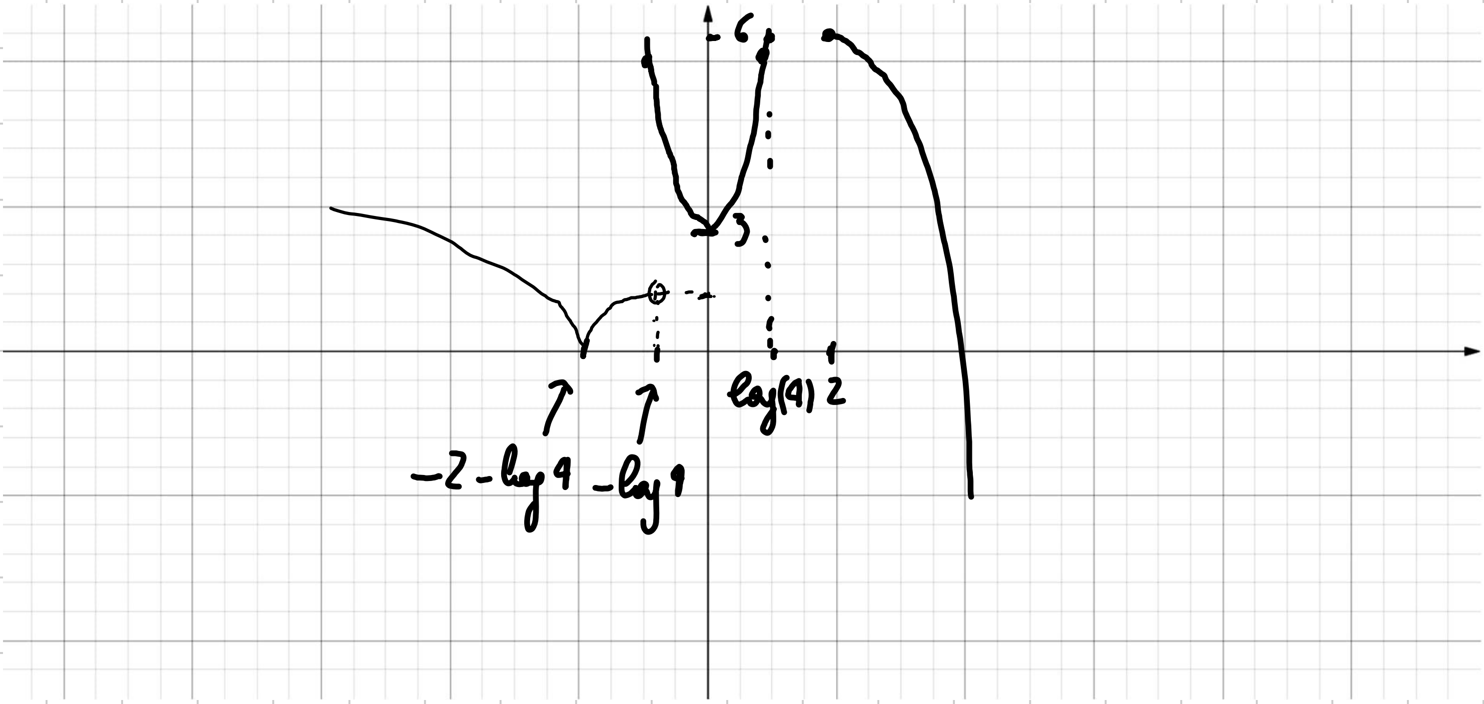
$$-2=\log y \iff y=e^{-2}<1$$

$$e^{-2}=\frac{1}{e^2}$$

$$\log(x-1)=\log e^{-2} \iff$$

$$x-1=e^{-2} \iff x=1+e^{-2}$$

Ex 2



$$f(x) = \begin{cases} \left|\sqrt[3]{x+\log 4+2}\right| & x < -\log 4 \\ e^{|x|}+2 & -\log 4 \le x \le \log 4 \\ 7-e^{x-2} & x \geq 2 \end{cases}$$

dom f

$x < -\log 4$ $\quad \sqrt[3]{}$ DEFINITA $\forall x \in \mathbb{R}$

$|x| \le \log 4$ $\quad$ NO ALTRE CONDIZIONI

$\log 4$? $\quad e \simeq 2{,}7 \quad e^2 > 4$

$$\log e < \log 4 < \log e^2$$

$$1 < \log 4 < 2$$

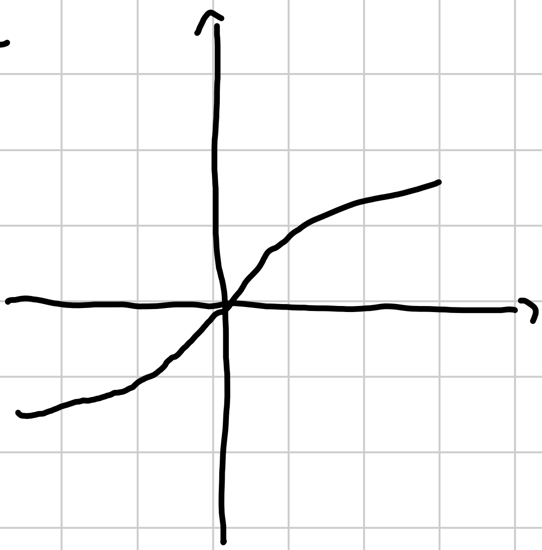
$x \geq 2$ NO ALTRE CONDIZIONI

$$\mathrm{Dom}\, f = (-\infty, \log 4) \cup [2, +\infty)$$

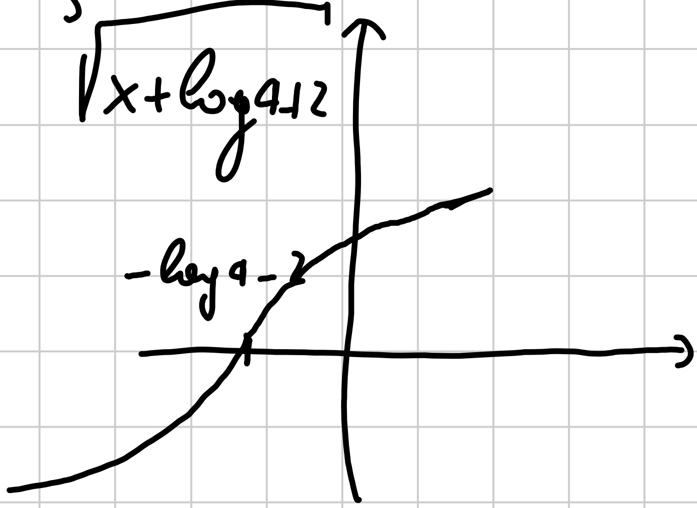
$x < -\log 4$

$$f(x) = \left|\sqrt[3]{x + \log 4 + 2}\right|$$

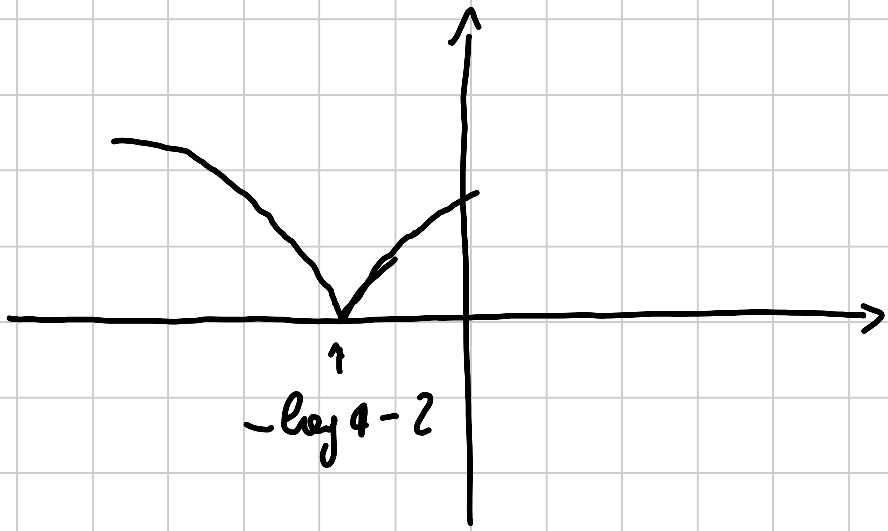
$\sqrt[3]{x}$



$\sqrt[3]{x+\log 4+2}$



$$\left|\sqrt[3]{x+\log 4+2}\right|$$

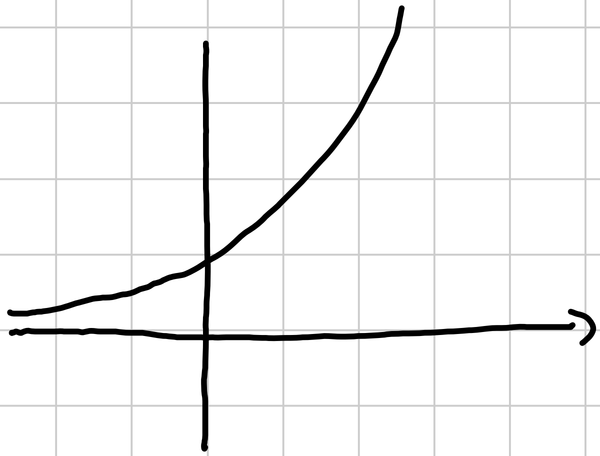


$$f(-\log 4) = \left|\sqrt[3]{2}\right| = \sqrt[3]{2}$$

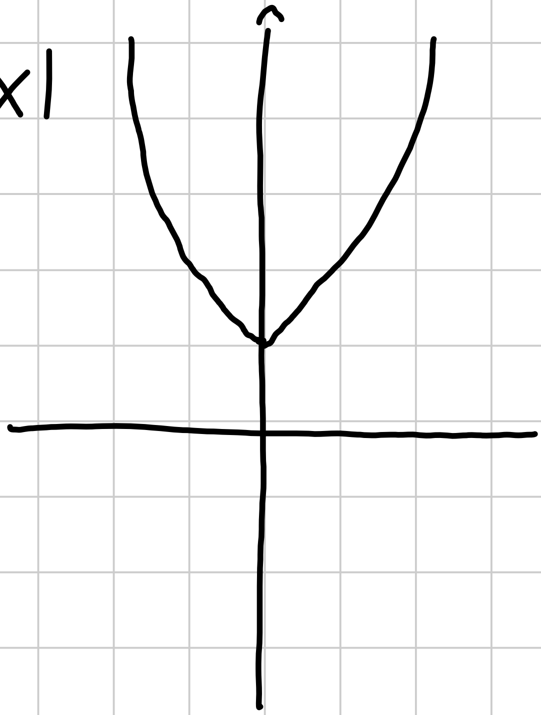
$$|x| \leq -\log 4$$

$$f(x) = e^{|x|} + 2$$

e^{x}



$e^{|x|}$



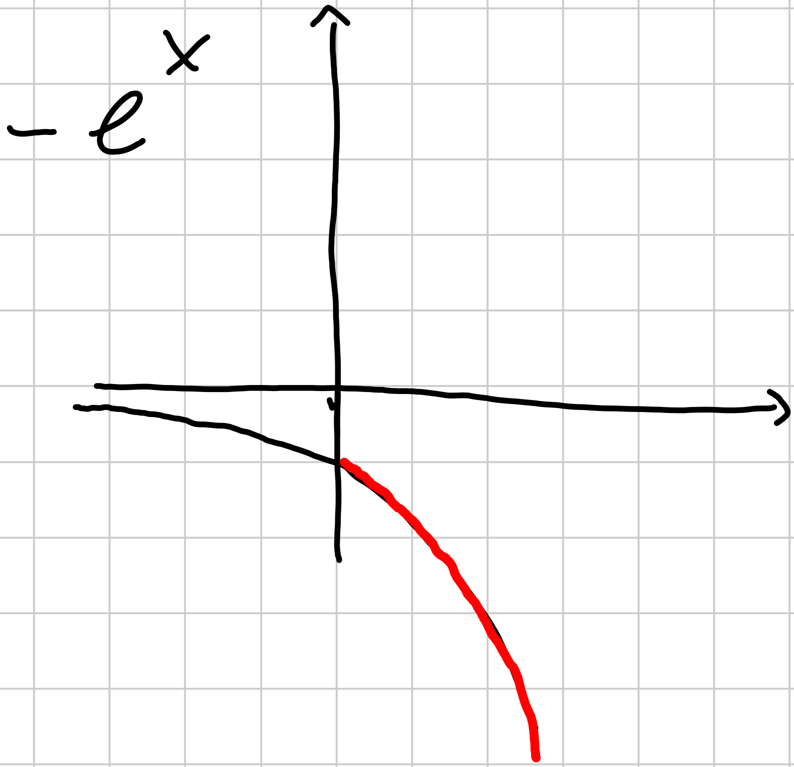
$$f(-\log 4) = e^{|-\log 4|} + 2 =$$

$$= \underbrace{e^{\log(4)}}_{4} + 2 = 6$$

$f(\log 4) = 6$ (FUNZIONE PARI)

$x \geq 2$ $f(x) = 7 - e^{x-2}$

$f(2) = 7 - e^{0} = 6$



$x \geq 2$

$\Rightarrow x - 2 \geq 0$

$-e^{(x-2)}$

EX TROVARE LE SOLUZIONI IN $\mathbb{R}$ DI:

1) $2x - 5\sqrt{x} + 2 = 0 \iff$

$5\sqrt{x} = 2x + 2 \iff \sqrt{x} = \frac{2x+2}{5}$

$\iff \begin{cases} x \geq 0 \\ \frac{2x+2}{\cancel{5}} \geq 0 \\ x = \frac{(2x+2)^2}{25} \end{cases} \quad \begin{cases} x \geq 0 \\ x \geq -1 \\ 25x = 4x^2 + 8x + 4 \end{cases}$

$\begin{cases} x \geq 0 \\ 4x^2 - 17x + 4 = 0 \end{cases} \qquad x_{1,2} = \frac{17 \pm \sqrt{289 - 64}}{8}$

$= \frac{17 \pm \sqrt{225}}{8} =$

$\iff \begin{cases} x \geq 0 \\ x = \frac{1}{4} \vee x = 4 \end{cases} \qquad = \frac{17 \pm 15}{8}$

ACCETTABILI

2) $\sqrt{4-x-x^2} = \sqrt{x^2+x} \iff$

$$\begin{cases} 4-x-x^2 \geq 0 \\ x^2+x \geq 0 \\ 4-x-x^2 = x^2+x \end{cases} \iff \begin{cases} x^2+x-4 \leq 0 \\ (x+1)x \geq 0 \\ 2x^2+2x-4=0 \end{cases}$$

$$\iff \begin{cases} x^2+x-4 \leq 0 \\ x \leq -1 \vee x \geq 0 \\ x^2+x-2=0 \end{cases} \iff \begin{cases} x^2+x-4 \leq 0 \\ x \leq -1 \vee x \geq 0 \\ (x+2)(x-1)=0 \end{cases}$$

$$\iff \begin{cases} x^2+x-4 \leq 0 & \longrightarrow \text{ok} \\ x \leq -1 \vee x \geq 0 & \longrightarrow \text{ok} \\ x=-2 \vee x=1 \end{cases}$$

$$(-2)^2+(-2)-4 = 4-2-4 = -2 \leq 0$$

$$(1)^2+1-4 = -2 \leq 0$$

3) $$\underbrace{\sqrt{x^4-x^2}}_{\geq 0}+\underbrace{x^2}_{\geq 0}=0 \iff$$

$$\iff \begin{cases}\sqrt{x^4-x^2}=0\\ x^2=0\end{cases} \iff \boxed{x=0}$$

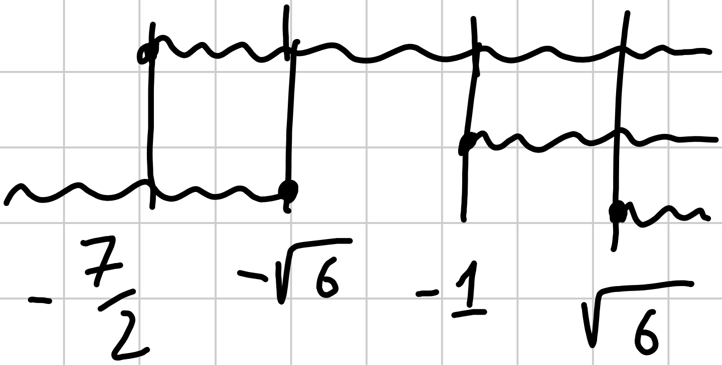
4) $$\sqrt{2x+7}\leq x+1 \iff$$

$$\begin{cases}2x+7\geq 0 \quad (\text{ESISTENZA RADICE})\\ x+1\geq 0\\ 2x+7\leq (x+1)^2\end{cases} \iff$$

$$\begin{cases}x\geq -\frac{7}{2}\\ x\geq -1\\ \cancel{2x}+7\leq x^2+\cancel{2x}+1\end{cases} \iff \begin{cases}x\geq -\frac{7}{2}\\ x\geq -1\\ x^2\geq 6\end{cases}$$

$$\iff \begin{cases} x \geq -\frac{7}{2} \\ x \geq -1 \\ x \leq -\sqrt{6} \lor x \geq \sqrt{6} \end{cases}$$

$2 < \sqrt{6} < 3$



$$Sol = [\sqrt{6}, +\infty)$$

5) $$\sqrt{\frac{1-x}{2+x}} \geq 0$$

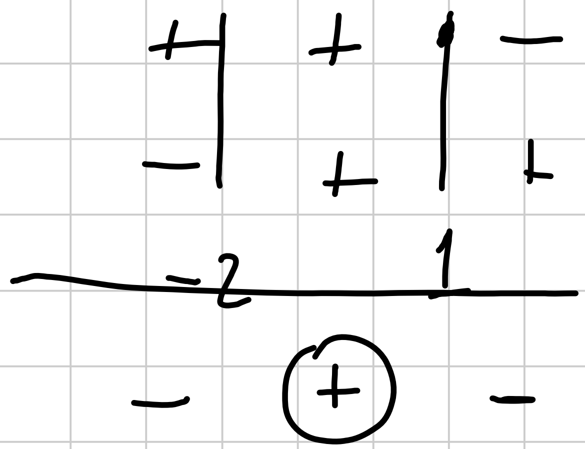
VERO $\forall x$ PER CUI E' DEFINITA LA RADICE

CE $\boxed{x \neq -2}$

$$\frac{1-x}{2+x} \geq 0$$

$N \geq 0 \iff x \leq 1$

$D > 0 \iff x > -2$



Sol.: $x \in (-2, 1]$

6) $$\frac{x-3}{1-\sqrt{x^2-1}} < 0$$

CE.

$\sqrt{x^2-1} \neq 1 \iff$

$$\begin{cases} x^2-1 \geq 0 \\ x^2-1 \neq 1 \end{cases} \iff$$

$$\begin{cases} x \leq -1 \vee x \geq 1 \\ \boxed{x \neq \pm\sqrt{2}} \end{cases}$$

N $x-3 \geq 0 \iff x \geq 3$

D $1-\sqrt{x^2-1} > 0$

$\iff \sqrt{x^2-1} < 1$

$$\iff \begin{cases} x \leq -1 \vee x \geq 1 \ \left(\text{Esist } \sqrt{\ }\right) \\ x^2-1 < 1 \iff x^2 < 2 \end{cases}$$

$$\iff \begin{cases} x \leq -1 \vee x \geq 1 \\ -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$$D > 0 \iff -\sqrt{2} < x \le -1 \quad \lor \quad 1 \le x < \sqrt{2}$$

CONCLUDERE COME ex

TROVARE LE SOLUZIONI IN $\mathbb{R}$ DI

$$|-x^2+4x+1| = 2x+2 \quad (\star) \iff$$

$$\begin{cases} 2x+2 \geq 0 \\ -x^2+4x+1 = 2x+2 \end{cases} \lor \begin{cases} 2x+2 \geq 0 \\ -x^2+4x+1 = -2x-2 \end{cases}$$

$$\iff \begin{cases} x \geq -1 \\ x^2 - 2x + 1 = 0 \end{cases} \vee \begin{cases} x \geq -1 \\ x^2 - 6x - 3 = 0 \end{cases}$$

$$\iff ① \begin{cases} x \geq -1 \\ (x-1)^2 = 0 \end{cases} \vee ② \begin{cases} x \geq -1 \\ x^2 - 6x - 3 = \end{cases}$$

$$|x-1| + |x+2| > 3$$

SI STUDIA CON METODO DEI CASI

Sol: ① $\boxed{x = 1}$

② $x_{1,2} = \dfrac{6 \pm \sqrt{36+12}}{2}$

$x_{1,2} = \dfrac{6 \pm \sqrt{48}}{2}$

$x_1 = 3 \pm 2\sqrt{3}$

$$② \begin{cases} x \geq -1 \\ x = 3 - 2\sqrt{3} \vee \\ x = 3 + 2\sqrt{3} \end{cases}$$

$$3 - 2\sqrt{3} \geq -1 \iff 4 \geq 2\sqrt{3} \iff$$

$2 \geq \sqrt{3}$ ok

Sln: 2 $x = 3 \pm 2\sqrt{3}$

Soln $\cancel{A}$ $x \in \{ 3 \pm 2\sqrt{3}, +1 \}$

Ex

$$|x^2 + x - 2| < 3x + 1 \iff$$

$$\begin{cases} 3x + 1 \geq 0 \\ -3x - 1 < x^2 + x - 2 < 3x + 1 \end{cases} \iff$$

$$\begin{cases} 3x + 1 \geq 0 \\ x^2 + x - 2 > -3x - 1 \\ x^2 + x - 2 < 3x + 1 \end{cases}$$

(Concludere x es)