Corso: ELEMENTI di MATEMATICA

Scheda di ESERCIZI N° 3

DISEQUAZIONI

1) DIS.ni di 1° e 2° grado

a) a1) $\frac{2}{3}(x-3) - \frac{5(2x-1)}{6} < \frac{2x-4}{2} + 2$ Risposta: ... $x > -\frac{7}{12}$

a2) $\frac{7x-3}{4} - \frac{1}{3}(2x-1) - \frac{1}{2} \leq \frac{3-2x}{6} - \frac{1}{4}$ Risposta: ... $x \leq \frac{14}{17}$

b) b1) $22 - 2x^2 + 10x < -5x^2 - 7x$ Risposta: ... $x \in \left]-\frac{11}{3}, -2\right[$

b2) $2 - 8x^2 \leq 0$ Risposta: ... $x \in \left]-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, +\infty\right[$

b3) $-5x^2 - 6x \geq 0$ Risposta: ... $x \in \left[-\frac{6}{5}, 0\right]$

b4) $3x^2 + 48 > 0$ Risposta: ... $\forall x \in \mathbb{R}$

b5) $5 - 6x^2 + 13x > 0$ Risposta: $x \in \left]-\frac{1}{3}, \frac{5}{2}\right[$

b6) $\left(\frac{3}{4}x - 1\right)^2 - x > 0$ Risposta: $x \in \left]-\infty, \frac{4}{9}\right[\cup \left]4, +\infty\right[$ es. 102) e 102bis) a pag. 74 degli ESERCIZI su ELLY

b7) $x(3x+2) > -\frac{1}{3}$ Risposta: $\forall x \neq -\frac{1}{3}$

b8) $9x^2 - 64x^4 > 0$ Risposta: $x \in \left]-\frac{3}{8}, 0\right[\cup \left]0, \frac{3}{8}\right[$

b9) $4x^3 - 20x^2 + 25x > 0$ Risposta: ... $x \in \left]0, \frac{5}{2}\right[\cup \left]\frac{5}{2}, +\infty\right[$

2) DIS.ni PRODOTTO, DIS.ni FRATTE

2a) $\left(\frac{9}{2} - \frac{3}{2}x + 7x^2\right) \cdot \left(\frac{4-5x}{6} - \frac{1}{8} - 6x\right) \cdot (9x^2+4) < 0$ Risposta: ... $x \in \left]\frac{13}{164}, +\infty\right[$

2b) $\left(\frac{3x-9}{4x+1} - 2\right) \cdot (x+1) \leq 0$ Risposta: ... $x \in \left[-\frac{11}{5}, -1\right] \cup \left]-\frac{1}{4}, +\infty\right[$

2c) $\frac{x^2+2x}{x^2-9} > 0$ Risposta ... $x \in \left]-\infty, -3\right[\cup \left]-2, 0\right[\cup \left]3, +\infty\right[$

2d) $\frac{4x^2-9}{x^2+2x+1} < 0$ Risposta ... $x \in \left]-\frac{3}{2}, -1\right[\cup \left]-1, \frac{3}{2}\right[$

2e) $\frac{6(x+1)}{9-x^2}+\frac{2}{x+3}>\frac{3}{3-x}$ Risposta... $x\in\,]-\infty,-3[\,\cup\,]-3,3[$

2f) $\left(\frac{4}{3x}-\frac{1}{9}\right)\left(\frac{3x+24}{4-x}-3\right)\leq 0$ Risposta... $x\in[-2,0[\,\cup\,]4,12]$

2g) $\frac{(3x-4)(x+5)}{x^2+3x-10}\geq 0$ Risposta... $x\in\,]-\infty,-5[\,\cup\,]-5,\frac{4}{3}]\,\cup\,]2,+\infty[$

2h) $\left(\frac{x^2}{3}+\frac{4}{3}x-7\right)\cdot\left(8x(2x+1)+1\right)>0$ Risposta... $x\in\,]-\infty,-7[\,\cup\,]3,+\infty[$

3) SISTEMI di DIS.^{ni} (o EQ.^{ni}), INSIEMI definiti tramite DIS.^{ni}

a)
a1) $\begin{cases}2x^2+2x=3+x^2\\ x(x+2)=6+x\end{cases}$ Risposta: $x=-3$

a2) $\begin{cases}2(x-1)-\frac{1}{3}(1-4x)\geq\frac{3x-1}{6}\\ -3(-x-1)-(2-x)\leq 4\end{cases}$ Risposta: $\emptyset$

a3) $\begin{cases}2(x-2)-(1-x)<x+1\\ \frac{2x+1}{5}-\frac{3x-2}{3}<\frac{1-2x}{6}\end{cases}$ Risposta: $x\in\,]\frac{21}{8},3[$ es. 77) pag. 50-51 degli ESERCIZI su ELLY

a4) $\begin{cases}\frac{300-2400x}{1000}<\frac{28x+84}{84}\\ \frac{3x^2+\frac{49}{3}-14x}{x-1+12x^2}>0\end{cases}$ Risposta: $x\in\,]\frac{1}{4},\frac{7}{3}[\,\cup\,]\frac{7}{3},+\infty[$

b) $A=\{x\in\mathbb{R}:10-2x^2+10x<-5x^2-3x\}$
$B=\left\{x\in\mathbb{R}:\left(\frac{3x-6}{4x+5}-2\right)(x+2)\leq 0\right\}.$

es. 65) pag. 43 degli ESERCIZI su ELLY

Determinate A, B, $A\cup B$, $A\cap B$, $A\setminus B$, $B\setminus A$.

$\boxed{\text{QUIZ}}$

Q1- Sia $a<0$, per quali valori risulta $\frac{3x+1}{2a}>0$?

A) nessun x B) $[-\frac{1}{3},+\infty)$ C) $(-\frac{1}{3},+\infty)$ D) $(-\infty,-\frac{1}{3})$ E) $(-\infty,-\frac{1}{3}]$

Q2- Un numero h è tale che $2<h<3$. Allora:

A) $h^2<4$ B) $\frac{1}{h^2}<\frac{1}{4}$ C) $\frac{1}{\sqrt{h}}<\frac{1}{3}$ D) $\sqrt{h}>2$ E) $\frac{1}{h}>\frac{1}{2}$

■ Esercizi da pag 12 a pag. 22 (fino n° XX) compresi QUIZ p. 21-22 - DISPENSA di ESERCIZI su ELLY

Scheda N.3 - SOLUZIONE

es. 1) a) a1) $4(x-3)-5(2x-1) < 3(2x-4)+12$

$-12x < 7 \qquad \boxed{x > -\frac{7}{12}} \qquad x \in \left]-\frac{7}{12}, +\infty\right[$

a2) $3(7x-3)-4(2x-1)-6 \leq 2(3-2x)-3$

$17x \leq 14 \qquad \boxed{x \leq \frac{14}{17}} \qquad x \in \left]-\infty, \frac{14}{17}\right]$

b) b1). $3x^2+17x+22<0 \qquad x_{1,2} = \frac{-17 \pm 5}{6}$ $\nearrow x_1 = -\frac{11}{3}$ $\searrow x_2 = -2$

SOL.ni $\boxed{x \in \left]-\frac{11}{3}, -2\right[}$

b2). $x^2 \geq \frac{1}{4}$ $\boxed{\begin{array}{c} x \in \left]-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, +\infty\right[\\ x \leq -\frac{1}{2} \text{ o } x \geq \frac{1}{2} \end{array}}$

b3). $5x^2+6x \leq 0 \qquad x(5x+6) \leq 0 \qquad x_1=0 \quad x_2 = -\frac{6}{5}$

$\boxed{x \in \left[-\frac{6}{5}, 0\right]}$

b4). $3x^2+48>0$ sempre vera $\boxed{\forall x \in \mathbb{R}}$

(essendo $x^2 \geq 0 \ \forall x \Rightarrow 3x^2+48 \geq 48 > 0 \ \forall x$)

b5). $6x^2-13x-5<0 \qquad x_{1,2} = \frac{13 \pm 17}{12}$ $\nearrow x_1 = -\frac{1}{3}$ $\searrow x_2 = \frac{5}{2}$

$\boxed{x \in \left]-\frac{1}{3}, \frac{5}{2}\right[}$

b6). $\frac{9}{16}x^2 - \frac{5}{2}x + 1 > 0 \qquad \frac{9}{8}x^2 - 5x + 2 > 0 \qquad x_{1,2} = \frac{5 \pm 4}{9/4} = \frac{4}{9}(5 \pm 4)$

$x_1 = \frac{4}{9} \quad x_2 = 4$

$\boxed{x \in \left]-\infty, \frac{4}{9}\right[\cup \left]4, +\infty\right[}$

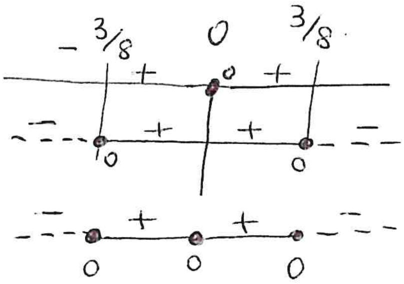
b7). $9x^2+6x+1>0 \qquad (3x+1)^2 > 0 \quad \forall x \neq -\frac{1}{3}$

b8). $x^2(9-64x^2)>0$

$F_1 = x^2 > 0 \quad \forall x \neq 0 \qquad F_1 = 0$ se $x = 0$

$F_2 = 9-64x^2 > 0 \quad x^2 < \frac{9}{64} \quad x \in \left]-\frac{3}{8}, \frac{3}{8}\right[$

$F_2 = 0 \quad x = \pm\frac{3}{8}$



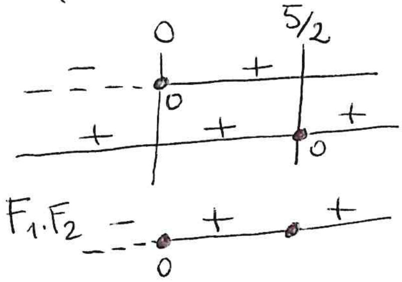
$F_1 \cdot F_2 > 0$ per $x \in \left]-\frac{3}{8}, 0\right[\cup \left]0, \frac{3}{8}\right[$

b9). $x(4x^2-20x+25)>0 \qquad x\cdot(2x-5)^2>0$

$F_1 = x > 0 \qquad F_1 = 0 \quad x = 0$

$F_2 = (2x-5)^2 > 0 \quad \forall x \neq \frac{5}{2}$

$F_2 = 0$ se $x = 5/2$



$F_1 \cdot F_2 > 0$ se $x \in \left]0, \frac{5}{2}\right[\cup \left]\frac{5}{2}, +\infty\right[$

2) 2a) $F_1 = \frac{9}{2} - \frac{3}{2}x + 7x^2 > 0 \quad \forall x \in \mathbb{R}$

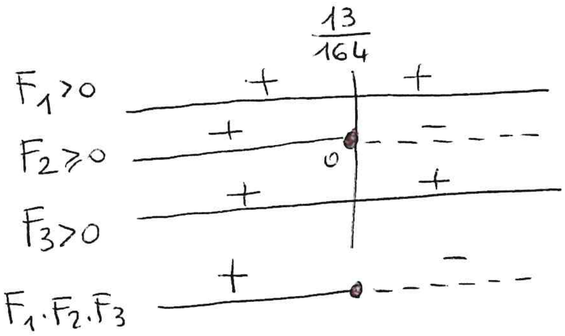
$7x^2 - \frac{3}{2}x + \frac{9}{2} > 0 \qquad (\Delta < 0, \cup)$

$F_2 \quad 4(4-5x) - 3 - 144x \geq 0$

$164x \leq 13 \qquad x \leq \frac{13}{164}$

$F_3 = 9x^2 + 4 > 0 \quad \forall x \in \mathbb{R}$

$(9x^2 + 4 \geq 4 \quad \forall x)$



$x \in \left]\frac{13}{164}, +\infty\right[$

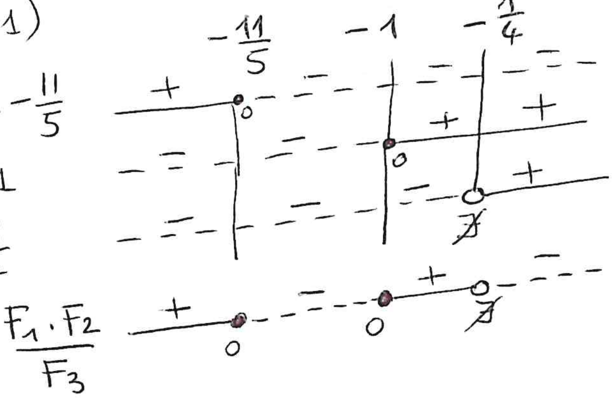
2b) $\left(\frac{3x-9}{4x+1} - 2\right)(x+1) = \frac{(-5x-11)}{(4x+1)} \cdot (x+1) \leq 0$

C.E. $x \neq -\frac{1}{4}$

$F_1 = (-5x-11) \geq 0 \quad x \leq -\frac{11}{5}$

$F_2 = x+1 \geq 0 \quad x \geq -1$

$F_3 = 4x+1 > 0 \quad x > -\frac{1}{4}$



$\frac{F_1 \cdot F_2}{F_3} \leq 0$ se $x \in \left[-\frac{11}{5}, -1\right] \cup$

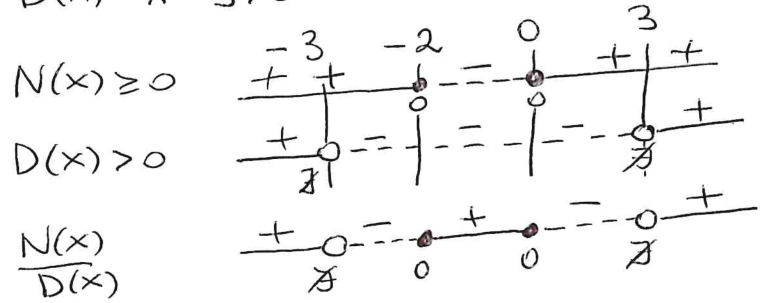
$\cup \left]-\frac{1}{4}, +\infty\right[$

2c) • $\dfrac{x^2+2x}{x^2-9} > 0$ C.E. $x \neq \pm 3$

$N(x) = x^2+2x \geq 0 \quad x(x+2) \geq 0 \quad x \in]-\infty, -2] \cup [0, +\infty[$

$N(x) = 0$ se $x = 0$ o $x = -2$

$D(x) = x^2 - 9 > 0 \quad x^2 > 9 \quad x < -3$ o $x > 3$



$\dfrac{N(x)}{D(x)} > 0 \quad x \in]-\infty, -3[\cup]-2, 0[\cup]3, +\infty[$

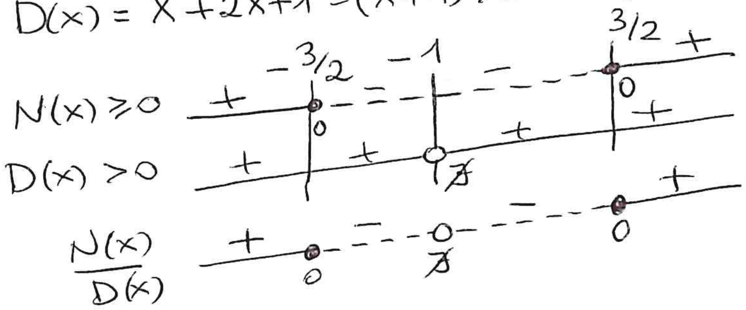
$\dfrac{4x^2-9}{x^2+2x+1} < 0$

C.E. $x \neq -1$

2d) • $N(x) = 4x^2 - 9 \geq 0 \quad x^2 \geq \frac{9}{4} \quad x \in]-\infty, -\frac{3}{2}] \cup [\frac{3}{2}, +\infty[$

$N(x) = 0$ se $x = \pm \frac{3}{2}$

$D(x) = x^2+2x+1 = (x+1)^2 > 0 \quad \forall x \neq -1$



$\dfrac{N(x)}{D(x)} < 0 \quad x \in]-\frac{3}{2}, -1[\cup]-1, \frac{3}{2}[$

Sol.ne Scheda N.3

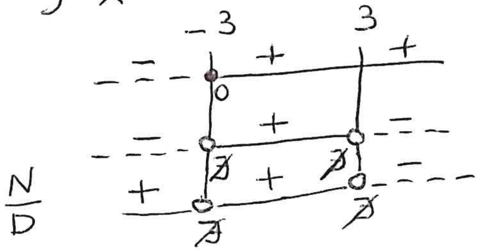
Continua es. 2

2e) $\dfrac{6(x+1)+2(3-x)-3(3+x)}{9-x^2}>0$ $\qquad \dfrac{x+3}{9-x^2}>0$

C.E. $x \neq \pm 3$

$N \geq 0 \quad x+3 \geq 0 \quad x \geq -3$

$D>0 \quad x^2<9 \quad -3<x<3$



Sol.ni $x \in \,]-\infty,-3[\, \cup \,]-3,3[$

2f) $\dfrac{(12-x)}{(9x)} \cdot \dfrac{3x+24-3(4-x)}{4-x} \leq 0$ $\qquad \dfrac{12-x}{9x} \cdot \dfrac{6x+12}{4-x} \leq 0$

C.E. $x \neq 0$, $x \neq 4$

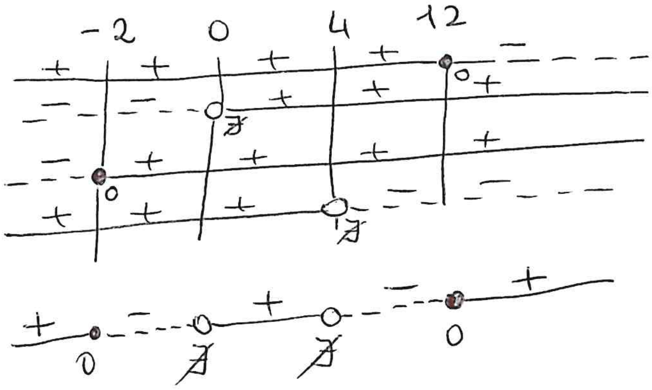
$N_1 = 12-x \geq 0 \quad x \leq 12$

$D_1 = 9x > 0 \quad x>0$

$N_2 = 6x+12 \geq 0 \quad x \geq -2$

$D_2 = 4-x>0 \quad x<4$

$\dfrac{N_1}{D_1} \cdot \dfrac{N_2}{D_2}$



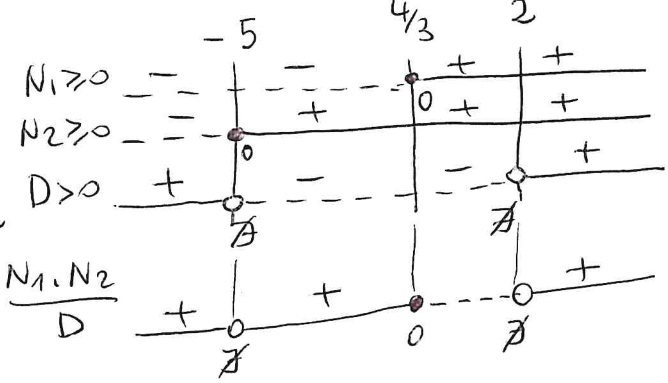
Sol.ni: $-2 \leq x < 0$ o $4 < x \leq 12$

$x \in [-2,0[\, \cup \,]4,12]$

2g) $N_1 = (3x-4) \geq 0 \quad x \geq \frac{4}{3}$

$N_2 = x+5 \geq 0 \quad x \geq -5$

$D = x^2+3x-10>0 \quad x<-5$ o $x>2$



C.E. $x^2+3x-10 \neq 0$

$x \neq -5 \quad x \neq 2$

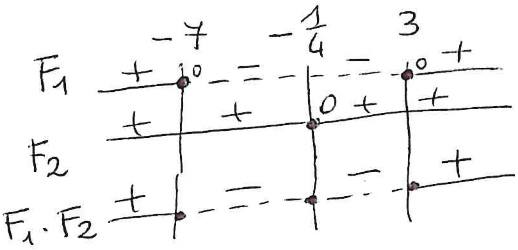
Sol.ni $x \in \,]-\infty,-5[\, \cup \,]-5,\frac{4}{3}] \, \cup \,]2,+\infty[$

es. 2) 2h) $\left(\frac{x^2}{3}+\frac{4}{3}x-7\right)\cdot\left(8x(2x+1)+1\right)>0$

$F_1(x)=\frac{1}{3}(x^2+4x-21)\geq 0 \quad x\leq -7 \ \circ\ x\geq 3$

$x^2+4x-21=0 \quad x_{1,2}=\frac{-2\pm\sqrt{4+21}}{1}$

$x_{1,2}=-2\pm 5 \nearrow x_1=-7$
$\searrow x_2=3$



$F_2(x)=16x^2+8x+1\geq 0$
$=(4x+1)^2\geq 0$
$\forall x$ con $=0 \ x=-\frac{1}{4}$

$F_1\cdot F_2>0$ se $x\in\,]-\infty,-7[\,\cup\,]3,+\infty[$

es. 3) a1) $\begin{cases} 2x^2+2x=3+x^2 \\ x(x+2)=6+x \end{cases}$ $\begin{cases} x^2+2x-3=0 \\ x^2+2x-x-6=0 \\ x^2+x-6=0 \end{cases}$ $\begin{cases} (x-1)(x+3)=0 \\ (x-2)(x+3)=0 \end{cases}$

$\begin{cases} x=1 \ \text{o} \ x=-3 \\ x=2 \ \text{o} \ x=-3 \end{cases}$ $S=\{x=-3\}$

QUIZ Q1- $a<0$ $\quad \frac{3x+1}{2a}>0$ $\quad F_1=3x+1\geq 0 \ \ x\geq -\frac{1}{3}$ con $F_1=0$ se $x=-\frac{1}{3}$

$F_2=2a<0$ sempre

Essendo il denominatore sempre <0 la frazione è >0 solo se il Numeratore è <0 : $S=\{x<-\frac{1}{3}\}=\,]-\infty,-\frac{1}{3}[$

Q2- $2<h \Rightarrow h^2>4$ A) Falsa

$2<h \Rightarrow h^2>4 \Rightarrow \frac{1}{h^2}<\frac{1}{4}$ B) Vera

se fosse $\frac{1}{\sqrt{h}}<\frac{1}{3} \Rightarrow \sqrt{h}>3 \Rightarrow h>9$ C) Falsa

se fosse $\sqrt{h}>2 \Rightarrow h>4$ D) Falsa

se fosse $\frac{1}{h}>\frac{1}{2} \Rightarrow h<2$ E) Falsa.

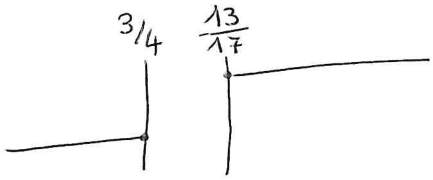
continua

Scheda 3 Sol.ne

-6-

es. 3) 22)
$$\begin{cases} 2(x-1)-\frac{1}{3}(1-4x)\geq\frac{3x-1}{6} \\ -3(-x-1)-(2-x)\leq 4 \end{cases} \qquad \begin{cases} 12x-12-2+8x-3x+1\geq 0 \\ 3x+3-2+x-4\leq 0 \end{cases}$$

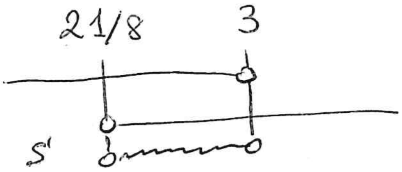
$$\begin{cases} 17x\geq 13 \\ 4x\leq 3 \end{cases} \qquad \begin{cases} x\geq\frac{13}{17} \\ x\leq\frac{3}{4} \end{cases}$$



$\frac{3}{4}\overset{?}{\geq}\frac{13}{17}\Leftrightarrow 51\geq 52$ Falso $\qquad S=\emptyset$

23)
$$\begin{cases} 2(x-2)-(1-x)<x+1 \\ \frac{2x+1}{5}-\frac{3x-2}{3}<\frac{1-2x}{6} \end{cases} \qquad \begin{cases} 2x-4-1+\cancel{x}<\cancel{x}+1 \\ 12x+6-30x+20-5+10x<0 \end{cases}$$

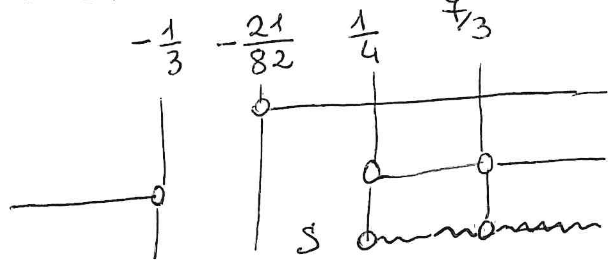
$$\begin{cases} 2x<6 \\ -8x+21<0 \end{cases} \qquad \begin{cases} x<3 \\ x>\frac{21}{8} \end{cases}$$



$S=\left]\frac{21}{8},3\right[$

$\frac{21}{8}<3=\frac{24}{8}$

24)
$$\begin{cases} \frac{300-2400x}{1000}<\frac{28x+84}{84} & \text{I}^a \\ \frac{3x^2+\frac{49}{3}-14x}{x-1+12x^2}>0 & \text{II}^a \end{cases}$$



$S=\left]\frac{1}{4},\frac{7}{3}\right[\cup\left]\frac{7}{3},+\infty\right[$

I^a semplificando 1°m per 100 e 2°m per 28

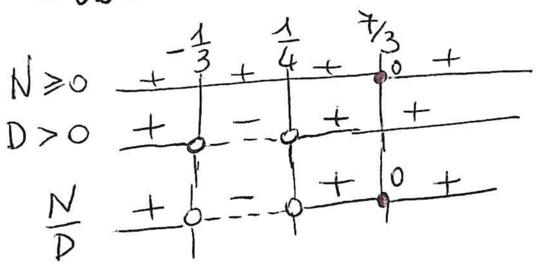
$\frac{3-24x}{10}<\frac{x+3}{3} \qquad \frac{9-72x-10x-30}{30}<0 \qquad -82x-21<0 \qquad x>-\frac{21}{82}$

II^a Dis.ne fratta $N=3x^2+\frac{49}{3}-14x\geq 0$

$9x^2-42x+49\geq 0$

$x_{1,2}=\frac{21\pm\sqrt{441-441}}{9}=\frac{21}{9}=\frac{7}{3}$

$N\geq 0\ \forall x$ con $N=0$ se $x=\frac{7}{3}$



II^a $\frac{N}{D}>0$ $x\in\left]-\infty,-\frac{1}{3}\right[\cup\left]\frac{1}{4},\frac{7}{3}\right[$
$\cup\left]\frac{7}{3},+\infty\right[$

$D=12x^2+x-1>0 \qquad x_{1,2}=\frac{-1\pm\sqrt{1+48}}{24}=$

$=\frac{-1\pm 7}{24}=$ $\nearrow x_1=-\frac{1}{3}$ $\searrow x_2=\frac{1}{4}$ $\qquad D>0$ se $x<-\frac{1}{3}$ o $x>\frac{1}{4}$

$-\frac{21}{82}\overset{?}{<}-\frac{1}{3} \qquad \frac{21}{82}>\frac{1}{3} \qquad 63>82$
Falso

es. 3) b) $A: \; 10 - 2x^2 + 10x < -5x^2 - 3x$

$$3x^2 + 13x + 10 < 0$$

$$x_{1,2} = \frac{-13 \pm \sqrt{169-120}}{6} = \frac{-13 \pm 7}{6} \;\begin{array}{l} \nearrow x_1 = -\frac{20}{6} = -\frac{10}{3} \\ \searrow x_2 = -\frac{6}{6} = -1 \end{array}$$

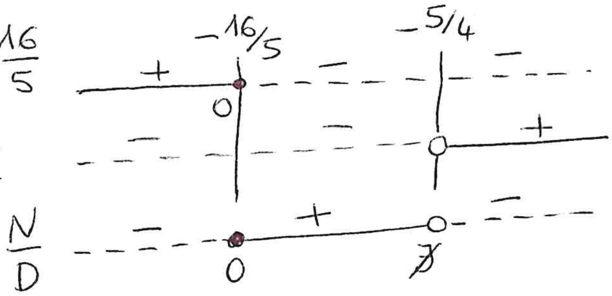
$A = \left]-\frac{10}{3}, -1\right[$

$B: \; \left(\frac{3x-6}{4x+5} - 2\right)(x+2) \leq 0$ Dis.^{ne} prodotto C.E. $4x+5 \neq 0$, $x \neq -5/4$

$F_1 = \frac{3x-6}{4x+5} - 2 \geq 0 \qquad \frac{3x-6-8x-10}{4x+5} \geq 0 \qquad \frac{-5x-16}{4x+5} \geq 0$

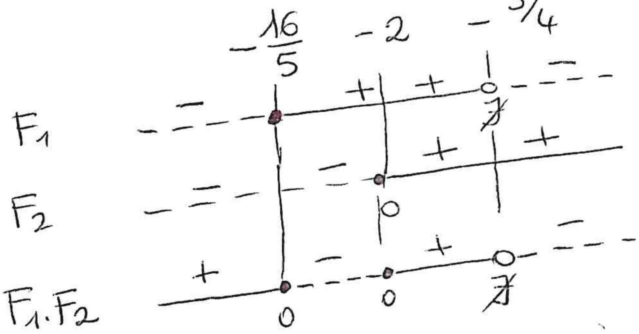
$N \geq 0 \quad -5x-16 \geq 0 \quad x \leq -\frac{16}{5}$

$D > 0 \quad 4x+5 > 0 \quad x > -\frac{5}{4}$



$F_1 \geq 0$ se $x \in \left[-\frac{16}{5}, -\frac{5}{4}\right[$

$F_2 = x+2 \geq 0 \iff x \geq -2$



$F_1 \cdot F_2 \leq 0$

$B = \left[-\frac{16}{5}, -2\right] \cup \left]-\frac{5}{4}, +\infty\right[$

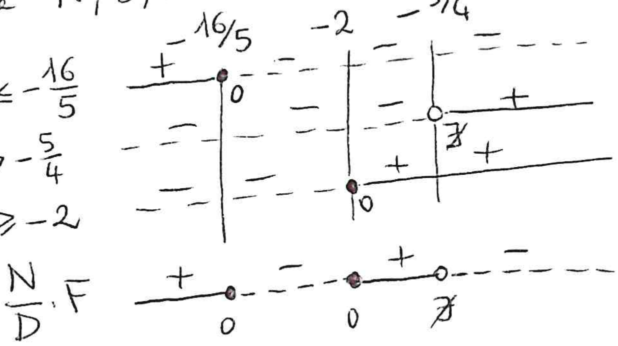
Si poteva anche considerare insieme N, D, F_2 :

$B: \; \frac{-5x-16}{4x+5} \cdot (x+2) \leq 0$

$N \geq 0 \quad x \leq -\frac{16}{5}$

$D > 0 \quad x > -\frac{5}{4}$

$F \geq 0 \quad x \geq -2$

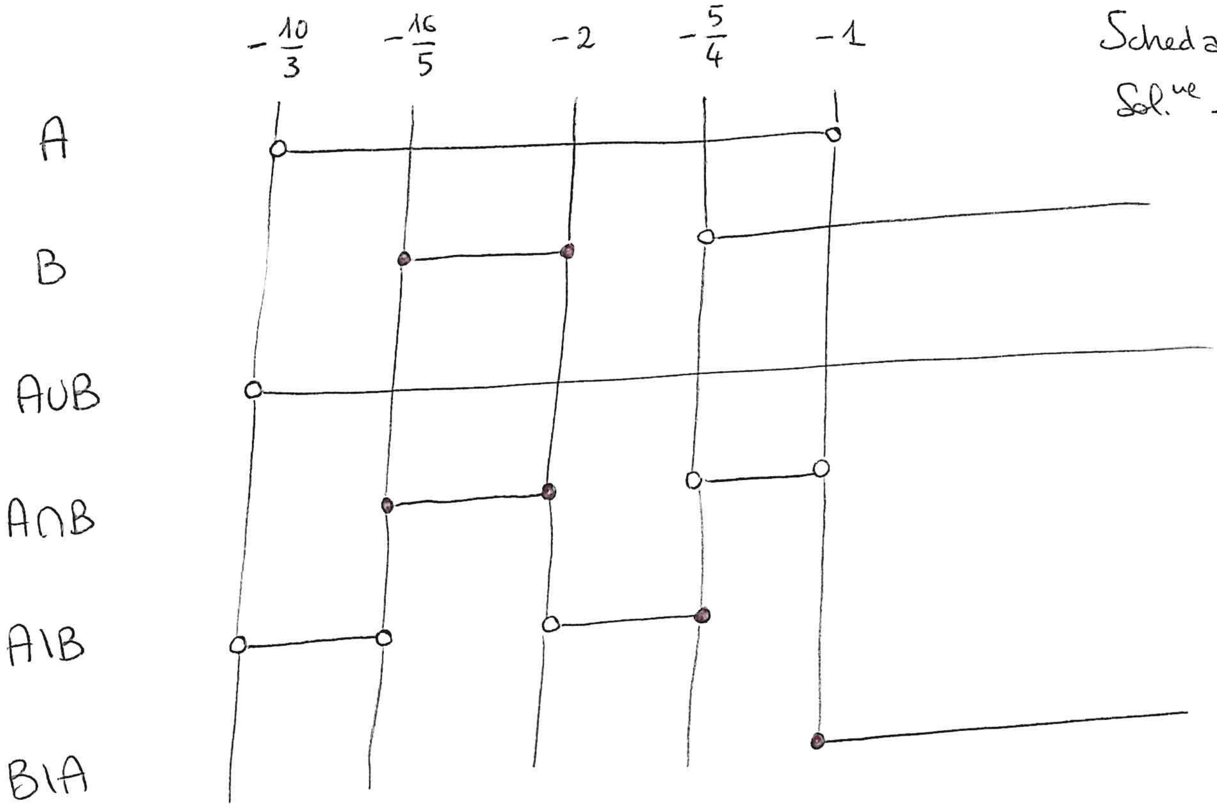


ottenendo più velocemente

$B = \left[-\frac{16}{5}, -2\right] \cup \left]-\frac{5}{4}, +\infty\right[$

Scheda 3

Sol.ne - 8 -



$-\frac{10}{3} < -\frac{16}{5} \iff \frac{10}{3} > \frac{16}{5} \iff 50 > 48$

$A \cup B = \left]-\frac{10}{3}, +\infty\right[$

$A \cap B = \left[-\frac{16}{5}, -2\right] \cup \left]-\frac{5}{4}, -1\right[$

$A \setminus B = \left]-\frac{10}{3}, -\frac{16}{5}\right[\cup \left]-2, -\frac{5}{4}\right]$

$B \setminus A = \left[-1, +\infty\right[$