Corso: ELEMENTI di MATEMATICA

SCHEDA N° 5 di ESERCIZI

ESPONENZIALI E LOGARITMI

0) Completate:

$6^{2} = 36 \qquad 3^{3} = 27 \qquad 2^{-5} = \frac{1}{32} \qquad \left(\frac{1}{3}\right)^{-4} = 81 \qquad 6^{-1} = \frac{1}{6}$

$5^{\cdots} = -\frac{1}{25} < 0$ impossibile $\qquad 10^{5} = 100'000 \qquad (-2)^{-3} = -\frac{1}{8} \qquad (-4)^{\cdots} = -\frac{1}{16}$ impossibile $-\frac{1}{16} = -\frac{1}{4^2}$ ma $(-4)^{-2} = \frac{1}{16}$

$(-3)^{\cdots} = -3^{-2}$ impossibile $(-3)^{-2} = \frac{1}{9}$

1) Risolvete:

$3^x = 27 \qquad 4^x = \frac{1}{64} \qquad e^x = e^3 \qquad 10^x = 0 \qquad e^x = -1 \qquad 10^x = \frac{1}{10000}$

$e^{-x} = \frac{1}{e^3} \qquad 5^{2x} = 5 \qquad 3^{2-5x} = \frac{1}{9} \qquad 4^{6x-5} = 4^{2+3x}$

$2^{1-x^2} = -2^2 \qquad e^{2x^2} = \frac{1}{e^{9x+5}}$.

sol.ne a pag. 3

$2^x = \frac{1}{64}$ Risposta: $x = -6$ $\qquad 5^x = 0$ Risposta: impossibile $5^x > 0 \ \forall x$

$10^{2x} = \frac{1}{1000}$ Risposta: $x = -\frac{3}{2}$ $\qquad 3^{4x} = -1$ Risposta: impossibile $3^{4x} > 0 \ \forall x$

$e^{-2x^2} = \frac{1}{e^4}$ Risposta: $x = \pm\sqrt{2}$ $\qquad 4^{1-3x} = 16$ Risposta: $x = -\frac{1}{3}$

$e^{4x-3} = e^{3+x}$ Risposta $x = 2$ $\qquad e^{4-x^2} = -e^2$ Risposta impossibile $e^x > 0$ sempre

$3^{\frac{7}{4}x} = (9)^{\frac{1}{4}x + \frac{1}{3}}$ Risposta $x = \frac{8}{15}$ $\qquad 5^{4x} = 125$ Risposta $x = \frac{3}{4}$

$10^{4x^2+4x} = \frac{1}{10}$ Risposta $x = -\frac{1}{2}$ $\qquad 10^{5x+1} = -2$ Risposta impos. $10^x > 0 \ \forall x$

$e^{2x^2} \cdot e^{x-\frac{7}{8}} = \frac{1}{e}$ Risposta $x = -\frac{1}{4}$ $\qquad e^x = 0$ Risposta impossibile $e^x > 0 \ \forall x$

$25 \cdot 5^{3x} = \frac{1}{(25)^{2x-\frac{1}{2}}}$ Risposta $x = -\frac{1}{7}$ $\qquad e \cdot e^{x^2} = \frac{e^{2x^2}}{e^3}$ Risposta $x = \pm 2$

2) Calcolate: Svolgim. a pag. 4

$\log_3 \frac{1}{3}$ $\log_4 64$ $\log_2 1$ $\log_{10} \frac{1}{100000}$ $\log_2(-2)$ $\log e^2$

$\log_4(-1)$ $\log_5 \frac{1}{125}$ $\log_{10} 100$ $\log e^{-2}$ $\log \frac{1}{e^3}$ $e^{\log 4}$.

$\log_3 0$ $\log_e 1$ $\log_e(-1)$ $\log_4 \frac{1}{16}$ $\log e^5$

$2e^{\log\frac{1}{3}}-1$ $(\log e^2)^{-3}$ $(3e^{-\log 2}-1)^2$

3) Risolvete

$\log_3 x = 81$ $\log_{10} x = -3$ $\log x = -1$ $\log_5 x = 0$

$\log_{10}(25x^2+36) = 2$ $\log_a \frac{1}{9} = -2$ $\log_{10} x = 4$

$\log x = 1$ $\log(3x^2) = 0$ (imponete la condizione sul C.E. cioè $3x^2>0$ e risolvetela)

$\log_2 x = 4$ $\log_{10} x = -6$

$e^x = 2$ $e^x = -1$ $\log x = 3$ $\log x = -4$

4) Calcolate utilizzando le proprietà dei logaritmi (specificando le proprietà utilizzate) Svolgim. a pag. 5

$\log_2 3 - \log_2 2 + \log_2 5 = \log_2 \frac{15}{2}$ $\log 7 + \log 6 - \log 3 = \log 14$

$\log \frac{1}{3} + 3\log 3 = 2\log 3$ $\log \frac{e}{4} = 1 - \log 4$

$3\log 4 - \log 8 - \log 1 = 3\log 2$

$7^{2+\log_7(\frac{1}{2})} = \frac{49}{2}$ $3^{-\log_3 4} = \frac{1}{4}$

$64^{\log_4 3} = 27$ $9^{2-\log_3 2} = \frac{81}{4}$ $25^{-\log_5 10} = \frac{1}{100}$

5) Calcolate utilizzando l'opportuno cambiamento di base:

$\log_4 7 \cdot \log_7 16 = 2$ $\log_2 3 \cdot \log_9 4 = 1$

ES. 1) $3^x = 27 \iff x=3$ $\quad 4^x = \frac{1}{64} \iff x=-3$ $\quad e^x = e^3 \iff x=3$

$10^x = 0$ impossibile $10^x > 0 \ \forall x$ $\quad e^x = -1$ impossibile $e^x > 0 \ \forall x$

$10^x = \frac{1}{10'000} = 10^{-4} \iff x=-4$ $\quad e^{-x} = \frac{1}{e^3} = e^{-3} \iff -x=-3 \iff x=3$

$5^{2x} = 5 \iff 2x=1 \iff x=\frac{1}{2}$ $\quad 3^{2-5x} = \frac{1}{9} = 3^{-2} \iff 2-5x=-2 \iff x=\frac{4}{5}$

$4^{6x-5} = 4^{2+3x} \iff 6x-5=2+3x \iff 3x=7 \iff x=\frac{7}{3}$

$2^{1-x^2} = -2^2 < 0$ IMPOSSIBILE $(2^x > 0 \ \forall x)$ $\quad e^{2x^2} = e^{-9x-5} \iff 2x^2=-9x-5$

$\iff 2x^2+9x+5=0 \iff x=\frac{-9-\sqrt{41}}{4}$ o $x=\frac{-9+\sqrt{41}}{4}$

$2^x = \frac{1}{64} = 2^{-6} \iff x=-6$ $\quad 10^{2x} = 10^{-3} \iff 2x=-3 \iff x=-\frac{3}{2}$

$e^{-2x^2} = e^{-4} \iff -2x^2=-4 \iff x^2=2 \iff x=\pm\sqrt{2}$

$4^{1-3x} = 4^2 \iff 1-3x=2 \iff 3x=-1 \iff x=-\frac{1}{3}$

$e^{4x-3} = e^{3+x} \iff 4x-3=3+x \iff 3x=6 \iff x=2$

$3^{\frac{7}{4}x} = (3^2)^{\frac{1}{4}x+\frac{1}{3}} \iff 3^{\frac{7}{4}x} = 3^{\frac{1}{2}x+\frac{2}{3}} \iff \frac{7}{4}x = \frac{1}{2}x+\frac{2}{3} \iff \frac{5}{4}x=\frac{2}{3} \iff x=\frac{8}{15}$

$5^{4x} = 5^3 \iff x=\frac{3}{4}$

$10^{4x^2+4x} = 10^{-1} \iff 4x^2+4x=-1 \iff 4x^2+4x+1=0 \iff (2x+1)^2=0$
$\iff x=-\frac{1}{2}$

$e^{2x^2+x-\frac{7}{8}} = e^{-1} \iff 2x^2+x-\frac{7}{8}=-1 \iff 2x^2+x+\frac{1}{8}=0$

$\iff x=-\frac{1}{4}$

$5^2 \cdot 5^{3x} = \frac{1}{(5^2)^{2x-\frac{1}{2}}} \iff 5^{2+3x} = \frac{1}{5^{4x-1}} = 5^{1-4x} \iff 2+3x=1-4x \iff$
$\iff 7x=-1 \iff x=-\frac{1}{7}$

$e^{1+x^2} = e^{2x^2-3} \iff 1+x^2=2x^2-3 \iff x^2=4 \iff x=\pm 2$

2) $\log_3 \frac{1}{3} = \log_3 3^{-1} = -1$ $\quad \log_4 64 = \log_4 4^3 = 3$

$\log_2 1 = 0$ $\quad \log_{10} 10^{-5} = -5$ $\quad \log_2(-2) \nexists$ $\quad \log_2 x$ è definito per $x>0$

$\log e^2 = \log_e e^2 = 2$ $\quad \log_4(-1) \nexists$ $\quad \log_5 5^{-3} = -3$ $\quad \log_{10} 10^2 = 2$

$\log e^{-2} = -2$ $\quad \log e^{-3} = -3$ $\quad e^{\log 4} = 4$ perchè $e^{\log x} = x \ \forall x>0$

$\log_3 0 \nexists$ $\quad \log_e 1 = 0$ $\quad \log_e(-1) \nexists$ $\quad \log_4 4^{-2} = -2$ $\quad \log e^5 = 5$

perchè $\log e^x = x \ \forall x \in \mathbb{R}$

$2e^{\log\frac{1}{3}} - 1 = 2 \cdot \frac{1}{3} - 1 = -\frac{1}{3}$

$(\log e^2)^{-3} = 2^{-3} = \frac{1}{8}$ $\qquad (3 \cdot e^{-\log 2} - 1)^2 = (3e^{\log\frac{1}{2}} - 1)^2 = (\frac{3}{2} - 1)^2 = \frac{1}{4}$

3) $\log_3 x = 81 \iff x = 3^{81}$!! volevo scrivere $\log_3 x = 4 \iff x = 3^4 = 81$!

$\log_{10} x = -3 \iff x = 10^{-3} = \frac{1}{1000}$ $\qquad \log x = -1 \iff x = e^{-1} = \frac{1}{e}$

$\log_5 x = 0 \iff x = 1$ $\qquad \log_{10}(\underbrace{25x^2+36}_{>0\ \forall x}) = 2 \iff 25x^2 + 36 = 10^2 = 100$

$\iff 25x^2 = 64 \iff x^2 = \frac{64}{25} \iff x = \pm\frac{8}{5}$

$\log_a \frac{1}{9} = -2 \iff a^{-2} = \frac{1}{9} \iff a = 3$

$\log_{10} x = 4 \iff x = 10^4 = 10'000$ $\qquad \log x = 1 \iff x = e$

$\log(3x^2) = 0 \iff \begin{cases} 3x^2 > 0 \\ 3x^2 = 1 \end{cases} \iff \begin{cases} x \neq 0 \\ x^2 = \frac{1}{3} \end{cases} \iff x = \pm\frac{\sqrt{3}}{3}$

$\log_2 x = 4 \iff x = 2^4 = 16$ $\qquad \log_{10} x = -6 \iff x = 10^{-6} = \frac{1}{1'000'000}$

$e^x = 2 \iff e^x = e^{\log 2} \iff x = \log 2$ $\qquad e^x = -1$ impossibile $e^x > 0 \ \forall x$

$\log x = 3 \iff x = e^3$ $\qquad \log x = -4 \iff x = e^{-4} = \frac{1}{e^4}$

4) $\log_2 3 - \log_2 2 + \log_2 5 = \log_2 \frac{3.5}{2} = \log_2 \frac{15}{2}$

$\log_a x + \log_a y = \log_a (xy) \quad x>0 \; y>0$

$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right) \quad x>0 \; y>0$

$\log 7 + \log 6 - \log 3 = \log \frac{42}{3} = \log 14$

$\log \frac{1}{3} + 3\log 3 = -\log 3 + 3\log 3 = 2\log 3$

$\log_a \frac{1}{x} = -\log_a x$

$\log\left(\frac{e}{4}\right) = \log e - \log 4 = 1 - \log 4$

$3\log 4 - \log 8 - \underbrace{\log 1}_{0} = 3\log 2^2 - \log 2^3 = 6\log 2 - 3\log 2 = 3\log 2$

$m\log_a b = \log_a b^m$

$7^{2+\log_7\left(\frac{1}{2}\right)} = 7^2 \cdot 7^{\log_7\left(\frac{1}{2}\right)} = 49 \cdot \frac{1}{2} = \frac{49}{2}$

$a^{x+y} = a^x \cdot a^y \qquad 7^{\log_7 x} = x \;\; \forall x>0$

$3^{-\log_3 4} = 3^{\log_3 \frac{1}{4}} = \frac{1}{4} \qquad 64^{\log_4 3} = (4^3)^{\log_4 3} = 4^{3\log_4 3} = 4^{\log_4 27} = 27$

$9^{2-\log_3 2} = \frac{9^2}{9^{\log_3 2}} = \frac{81}{(3^2)^{\log_3 2}} = \frac{81}{3^{2\log_3 2}} = \frac{81}{3^{\log_3 4}} = \frac{81}{4}$

$25^{-\log_5 10} = (5^2)^{-\log_5 10} = 5^{-2\log_5 10} = 5^{\log_5 \frac{1}{100}} = \frac{1}{100}$

5) $\log_4 7 \cdot \log_7 16 = \cancel{\log_4 7} \cdot \frac{\log_4 16}{\cancel{\log_4 7}} = \log_4 16 = \log_4 4^2 = 2$

$\log_2 3 \cdot \log_9 4 = \log_2 3 \cdot \frac{\log_2 4}{\log_2 9} = \log_2 3 \cdot \frac{\log_2 2^2}{\log_2 3^2} = \cancel{\log_2 3} \cdot \frac{\cancel{2}}{\cancel{2\log_2 3}} = 1$