es. 160)

i) $A = \{ x \in \mathbb{R} : \ x = \frac{\pi}{6} + 2k\pi \ (k \in \mathbb{Z}) \ \underline{o} \ x = \frac{5}{6}\pi + 2k\pi \ (k \in \mathbb{Z}) \}$

ii) $A = \{1\}$

iii) $A = \emptyset$

iv) $A = [-\frac{3}{2}, -1] \cup [1, +\infty[$ v) $A = [0,1[\cup [2,3[$

vi) $A =]-\infty, -2]$ vii) $A = [1,2]$

viii) $A =]0, \frac{1}{2}[\cup]1, \frac{3}{2}[$

161)

i) $A = \{-\frac{5}{6}, -\frac{1}{2}, \frac{7}{3}\}$

ii) $A =]-\infty, 2-\sqrt{3}] \cup]3, 2+\sqrt{3}]$

iii) $A =]\frac{1-\sqrt{17}}{2}, -1[\cup]\frac{1+\sqrt{17}}{2}, +\infty[$

iv) $A = [\frac{1}{\sqrt{5}}, 1]$

v) $A = [0,1[\cup]\frac{3+\sqrt{5}}{2}, +\infty[$

vi) $A =]-\frac{13}{3}, -3[\cup \{0\} \cup [\frac{1}{4}, +\infty[$

vii) $A = \bigcup_{k \in \mathbb{Z}}]\frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi[$

viii) $A = [0, \frac{\pi}{6}] \cup [\frac{5}{6}\pi, \frac{13}{6}\pi)$

ix) $A =]-5,-4] \cup [0,1[$

x) $A = [-5,-3] \cup [4,+\infty[$

xi) $A = [2/15, +\infty[$

xii) $A = [\frac{1}{e}, 1]$

xiii) $A = [0,2]$

xiv) $A = \{\frac{\pi}{6}, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi\}$

xv) $A =]-\infty, -3] \cup [1,3]$

xvi) $A = [\frac{\pi}{4}, \frac{5}{4}\pi]$

162) $A =]\frac{1}{2}, +\infty[$ $\quad A \cap [-7,4] =]\frac{1}{2}, 4]$

$B =]-2,0[\cup]1,3[$ $\quad B \setminus ([-3,-1[) = [-1,0[\cup]1,3[$

163) i) $A = [3,+\infty[$ $\quad B = [-2,4]$ $\quad A \cup B = [-2,+\infty[$ $\quad A \cap B = [3,4]$

$A \setminus B =]4,+\infty[$ $\quad B \setminus A = [-2,3[$

ii) $A =]-\infty,1[\cup]7,+\infty[$ $\quad B = [-2,1] \cup [3,+\infty[$

$A \cup B =]-\infty,1] \cup [3,+\infty[$ $\quad A \cap B = [-2,1[\cup]7,+\infty[$

$A \setminus B =]-\infty,-2[$ $\quad B \setminus A = \{1\} \cup [3,7]$

iii) $A =]-4,-2[\cup]0,2[$ $\quad B =]-3,+\infty[$

$A \cup B =]-4,+\infty[$ $\quad A \cap B =]-3,-2[\cup]0,2[$

$A \setminus B =]-4,-3]$ $\quad B \setminus A = [-2,0] \cup [2,+\infty[$

164) i) $A = \{-\frac{4}{5}, 2\}$ $\quad A \cup [1,+\infty[= \{-\frac{4}{5}\} \cup [1,+\infty[$

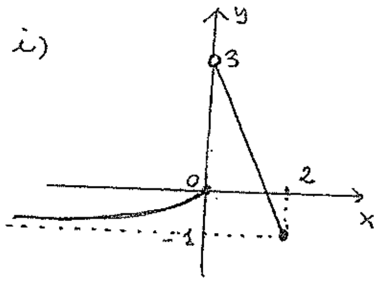
ii) $A = [\frac{1}{2},+\infty[$ $\quad A \setminus [3,+\infty[= [\frac{1}{2},3[$

iii) $A = [-2,0] \cup [4,6]$ $\quad B =]\frac{3}{2},+\infty[$

$A \setminus B = [-2,0]$ $\quad B \setminus A =]\frac{3}{2},4[\cup]6,+\infty[$ $\quad A \cap B = [4,6]$

iv) $A = [-\frac{\pi}{3}, \frac{\pi}{3}]$

165) i)



$\operatorname{Im} f = [-1,3[$

$-1 = \min_{]-\infty,2]} f = f(2)$

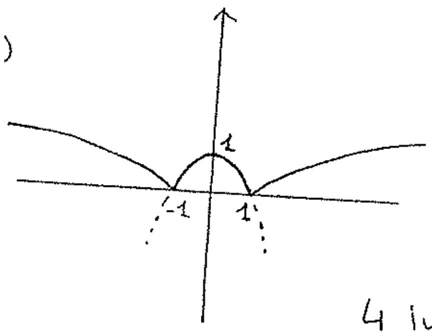
$\nexists \max f$

$\boxed{f(x) = \frac{1}{e} - 1}$ $\quad e^x = \frac{1}{e} = e^{-1}$

$\boxed{x = -1}$

2 intervalli di monotonia: $]-\infty,0] \nearrow$ $]0,2] \searrow$

ii)



$\operatorname{Im} f = [0,+\infty[$

$0 = \min_{\mathbb{R}} f = f(-1) = f(1)$

$\max_{\mathbb{R}} f = \nexists$

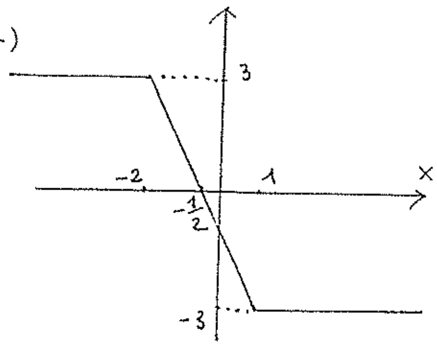
$\boxed{f(x) > 2}$ $\quad \log|x| = 2 = \log e^2$, $x \neq 0$

$|x| = e^2$ $\quad x = \pm e^2$

$\boxed{x < -e^2 \text{ o } x > e^2}$

4 intervalli di monotonia: $]-\infty,-1] \searrow$ $[-1,0] \nearrow$ $[0,1] \searrow$ $[1,+\infty[\nearrow$

iii)



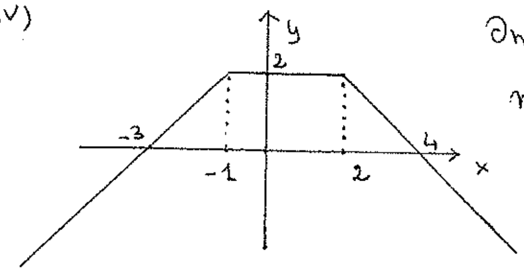
$\operatorname{Im} f = [-3,3]$

$-3 = \min f = f(x) \quad \forall x \geq +1$

$3 = \max f = f(x) \quad \forall x \leq -2$

f è debolmente decrescente su $\mathbb{R}$, ma non strettamente

iv)



$\operatorname{Im} f =]-\infty, 2]$

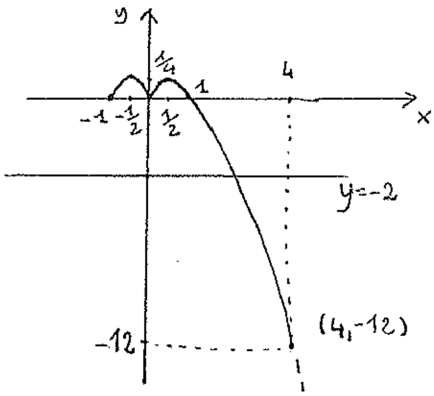
$\min_{\mathbb{R}} f = \nexists$

$2 = \max_{\mathbb{R}} f = f(x) \quad \forall x \; -1 \leq x \leq 2$

strett. crescente su $]-\infty,-1]$, debolmente crescente (ma non strett.) su $]-\infty,2]$

strett. decrescente su $[2,+\infty[$, debolmente decrescente (ma non strett.) su $[-1,+\infty[$

v)



$\operatorname{Im} f =]-12, \frac{1}{4}]$

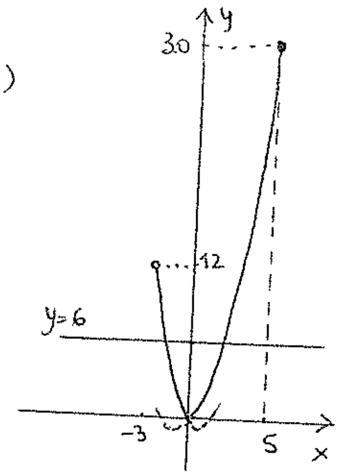
$-12 = \min_{[-1,4]} f = f(4)$

$\frac{1}{4} = \max_{[-1,4]} f = f(-\frac{1}{2}) = f(\frac{1}{2})$

$[-1,-\frac{1}{2}] \nearrow \; [-\frac{1}{2},0] \searrow \; [0,\frac{1}{2}] \nearrow \; [\frac{1}{2},4] \searrow$

$\boxed{f(x)=-2} \quad \overset{x \geq 0}{x(1-x)=-2} \quad x^2-x-2=0 \quad \boxed{x=2}$

vi)



$\operatorname{Im} f = [0,30]$

$0 = \min_{]-3,5]} f = f(0)$

$30 = \max_{]-3,5]} f = f(5)$

$]-3,0] \searrow \; [0,5] \nearrow$

$\boxed{f(x) \geq 6} \quad x<0 \quad f(x) = -x(1-x) = x^2-x$

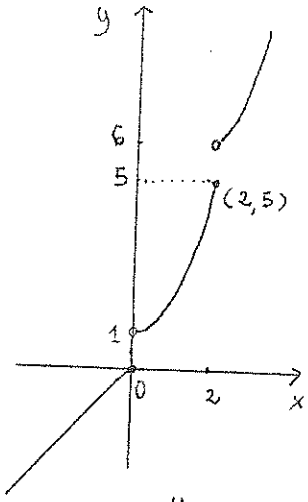
$x^2-x=6 \quad x=-2$

$x \geq 0 \quad f(x) = x(1+x) = x^2+x$

$x^2+x=6 \quad x=2$

$\boxed{x \in]-3,-2] \cup [2,5]}$

vii)



$\mathrm{Im} f =]-\infty,0] \cup]1,5] \cup]6,+\infty[$

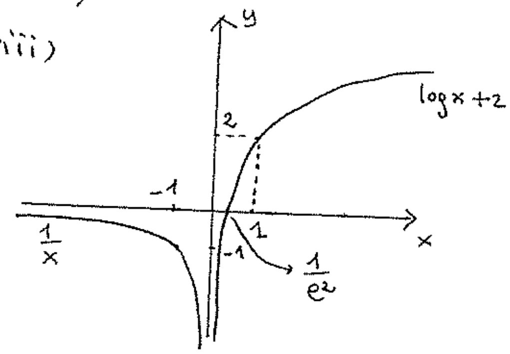
$\min f = \nexists \quad \max f = \nexists$

$y = e^{x-2} + 5 \quad y = e^x$ a destra di 2 e in alto di 5

$\boxed{f(x)=8} \quad e^{x-2}+5=8 \quad e^{x-2} = 3 = e^{\log 3} \quad \boxed{x = 2+\log 3}$

f è crescente su $\mathbb{R}$

viii)



$\mathrm{Im} f = \mathbb{R} \qquad \min f \nexists \quad \max f \nexists$

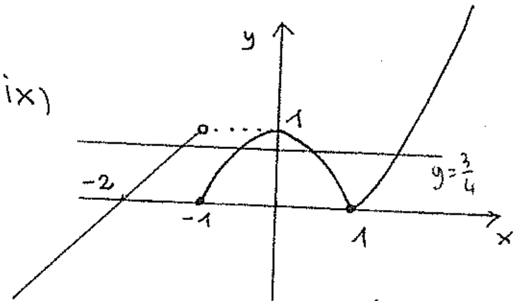
$f \searrow]-\infty,0[$

$f \nearrow]0,+\infty[$

$\boxed{f(x) \leq -1} \quad x<0 \quad \frac{1}{x} = -1 \quad x=-1 \quad x>0 \quad \log x + 2 = -1$

$\log x = -3 \quad x = e^{-3} = \frac{1}{e^3} \quad \boxed{x \in [-1,0[\cup]0, \frac{1}{e^3}]}$

ix)



$\mathrm{Im} f = \mathbb{R} \quad \min f = \nexists \quad \max f = \nexists$

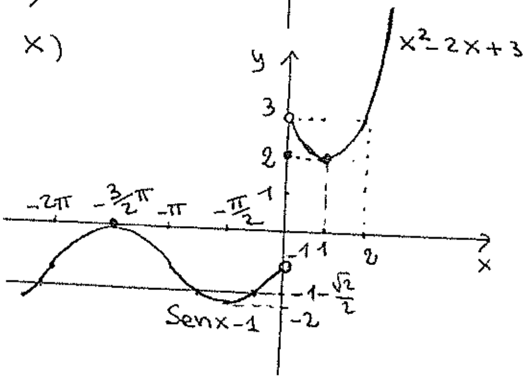
$f \nearrow$ su $]-\infty,-1[$, su $[-1,0]$ e su $[1,+\infty[$

$\searrow$ su $[0,1]$

$\boxed{f(x) = \frac{3}{4}}$ retta $x+2 = 3/4 \quad x = -\frac{5}{4}$ ($x<-1$) parabola $|-x^2+1| = 3/4$ ($x \geq -1$) (*)

$-x^2+1 = \frac{3}{4}$ o $-x^2+1 = -\frac{3}{4} \quad x^2 = \frac{1}{4}$ o $x^2 = \frac{7}{4}$

x)



$\mathrm{Im} f = [-2,0] \cup [2,+\infty[\quad \max f = \nexists$

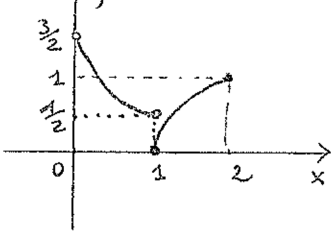
$\min_{\mathbb{R}} f = -2 = f(-\frac{\pi}{2}) = f(-\frac{5}{2}\pi) = f(-\frac{9}{2}\pi) = \dots$

$= f(-\frac{\pi}{2} + 2k\pi) \quad k \in \mathbb{Z}, k \leq 0$

$\boxed{f(x) = -1-\frac{\sqrt{2}}{2}} \quad \mathrm{sen}\, x = -\frac{\sqrt{2}}{2} \quad \boxed{\begin{array}{l} x = -\frac{\pi}{4} + 2k\pi \quad k \in \mathbb{Z} \; k \leq 0 \\ x = -\frac{3}{4}\pi + 2k\pi \quad k \in \mathbb{Z} \; k \leq 0 \end{array}}$

(*) $\begin{cases} x \geq -1 \\ x^2 = \frac{1}{4} \end{cases}$ o $\begin{cases} x \geq -1 \\ x^2 = \frac{7}{4} \end{cases} \iff \begin{cases} x \geq -1 \\ x = \pm\frac{1}{2} \end{cases}$ o $\begin{cases} x \geq -1 \\ x = \pm\frac{\sqrt{7}}{2} \end{cases}$ Sol.ni $\{x = -\frac{5}{4}, x = \pm\frac{1}{2}, x = \frac{\sqrt{7}}{2}\}$

xi)



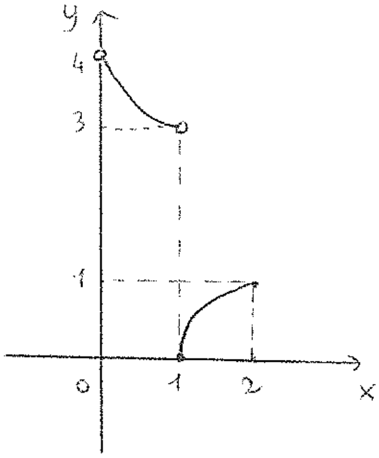
$\mathrm{Im} f = [0, \frac{3}{2}[\quad \min f = 0 = f(1) \quad f \searrow$ su $]0,1]$

$\max f \nexists \qquad \nearrow [1,2]$

$y = (x-1)^2 + \frac{1}{2}$ parabola verso l'alto

$V(1, \frac{1}{2})$

xii)



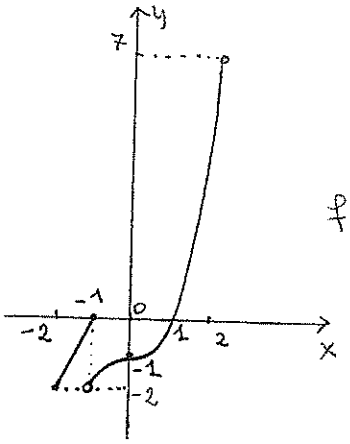
$y=(x-1)^2+3$ parabola verso l'alto di $V(1,3)$

$\mathcal{I}m f = [0,1] \cup]3,4[$ (è iniettiva)

$\min f = 0 = f(1)$

$\max f \nexists$ $\quad f \searrow$ su $]0,1]$ $\nearrow [1,2]$

xiii)



$\mathcal{I}m f = [-2,7]$

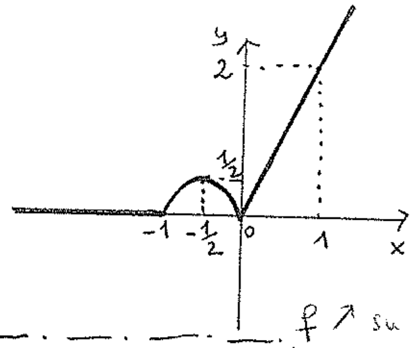
$\min f = -2 = f(-2)$

$\max f = 7 = f(2)$

$f \nearrow [-2,-1]$ e $]-1,2]$

(*)

xiv)



$\mathcal{I}m f = [0,+\infty[$

$\max f \nexists$

$\min f = 0 = f(x)$ $\forall x \in]-\infty,-1]$ e anche in $x=0$

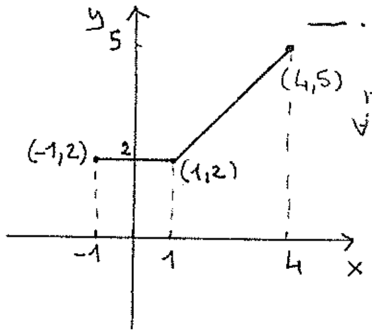
$f \nearrow$ su $[-1,-\frac{1}{2}]$ e su $[0,+\infty[$

$\searrow$ su $[-\frac{1}{2},0]$

DEBOLM CRESC su $]-\infty,-\frac{1}{2}]$

DEBOLM DECRESC su $]-\infty,-1]$

xv) $f(x)=\begin{cases} 2 & -1\le x\le 1 \\ x+1 & 1<x\le 4 \end{cases}$



$\max f = 5 = f(4)$

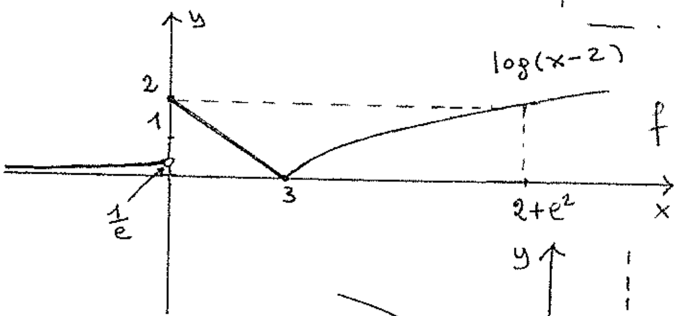
$\min f = 2 = f(x)$ $\forall x \in [-1,1]$

$\mathcal{I}m f = [2,5]$

$f \nearrow$ su $[1,4]$

DEBOLM CRESC su $[-1,4]$, DEB. $\downarrow$ su $[-1,1]$

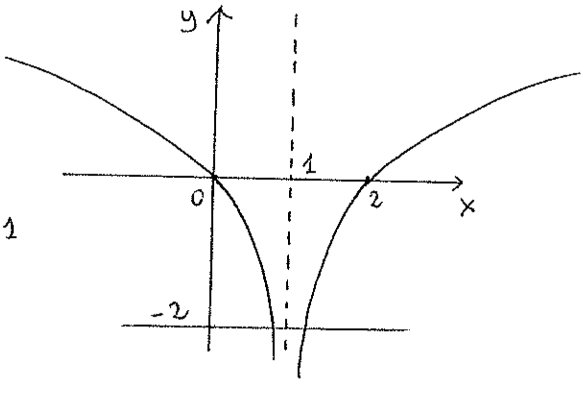
xvi)



$\mathcal{I}m f = [0,+\infty[$ $\quad \min f = 0 = f(3)$ $\quad \max f \nexists$

$f \nearrow]-\infty,0]$, $[3,+\infty[$ $\quad f \searrow [0,3]$ (**)

xvii)



$\mathcal{I}m f = \mathbb{R}$

$y=\log|x|$ a destra di 1

$\min f \nexists$ $\quad \max f \nexists$

$f \nearrow]1,+\infty[$ $\searrow]-\infty,1[$

$\boxed{f(x)<-2}$

$\log|x-1| = -2 = \log e^{-2} = \log \frac{1}{e^2}$

$x \ne 1$

$|x-1| = \frac{1}{e^2}$ $\quad x = 1 \pm \frac{1}{e^2}$

Sol.^{ni} $\boxed{x \in]1-\frac{1}{e^2},1[\cup]1,1+\frac{1}{e^2}[}$

(*) $\boxed{f(x)=-\frac{7}{8}}$ retta $y=2x+2=-\frac{7}{8}$ $\quad 2x=-\frac{23}{8}$ $\quad x=-\frac{23}{16}$

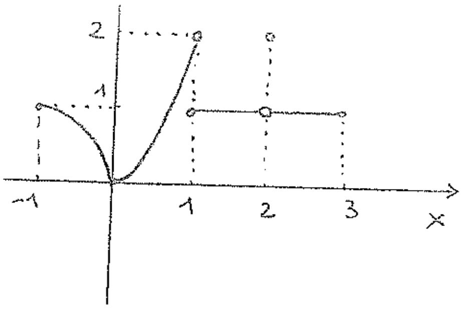
$y=x^3-1=-\frac{7}{8}$ $\quad x^3=\frac{1}{8}$ $\quad x=\frac{1}{2}$

Sol.^{ni} $\boxed{\left\{ x=-\frac{23}{8}, x=\frac{1}{2} \right\}}$

(**) $\boxed{f(x)\ge 2}$ $\quad 2=\log(x-2)$ $\quad \log e^2 = \log(x-2)$ $\quad x=2+e^2$

Sol.^{ni} $\boxed{\{x=0\} \cup [2+e^2,+\infty[}$

xviii)



$\Im m f = [0,2]$ $\min f = 0 = f(0)$ $\max f = 2 = f(2)$

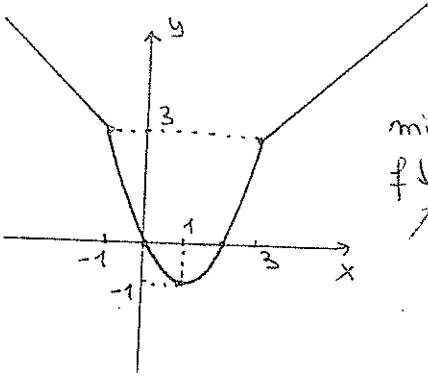
$f \nearrow$ su $[0,2[$ $\searrow$ su $[-1,0]$ DEBOLM. CRESC su $[1,2]$ e $]2,3]$

DEBOLM. DECRESC. su $[1,2[$ e $[2,3]$

xix)

$$f(x) = \begin{cases} \cancel{x} & x \geq 3 \\ x^2 - 2x & -1 \leq x < 3 \\ 2 - x & x < -1 \end{cases}$$

$|x+1| + |x-3| - 2 \neq 0 \quad \forall x$



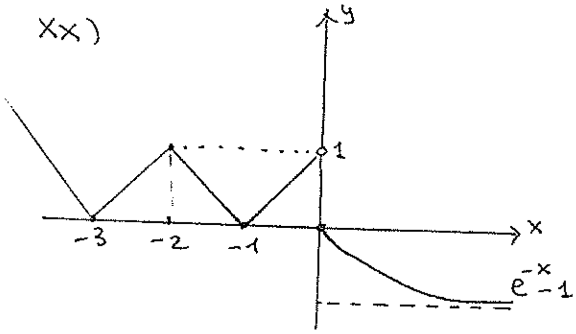
$\Im m f = [-1, +\infty[$

$\min f = -1 = f(1)$ $\max f \nexists$

$f \searrow$ su $]-\infty, 1]$

$\nearrow$ su $[1, +\infty[$

xx)



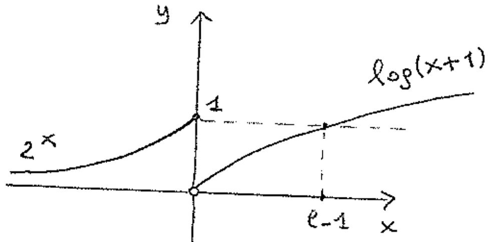
$\Im m f =]-1, +\infty[$

$\min f \nexists$ $\max f \nexists$

$f \nearrow$ su $[-3,-2]$, $[-1,0[$

$\searrow$ su $]-\infty,-3]$, $[-2,-1]$, $[0,+\infty[$

xxi)

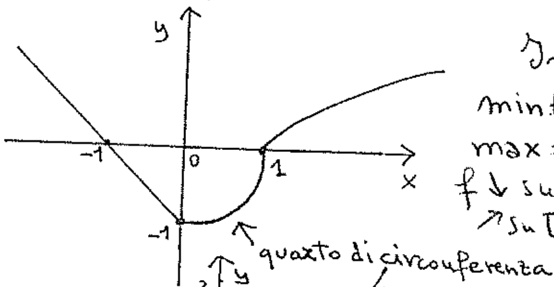


$\Im m f =]0, +\infty[$ $\min f \nexists$

$\max f \nexists$ $f \nearrow$ su $]-\infty, 0]$ e su $]0, +\infty[$

$\boxed{f(x) < 1}$ $\boxed{x \in\ ]-\infty, 0[\cup]0, e-1[}$

xxii)



$\Im m f = [-1, +\infty[$

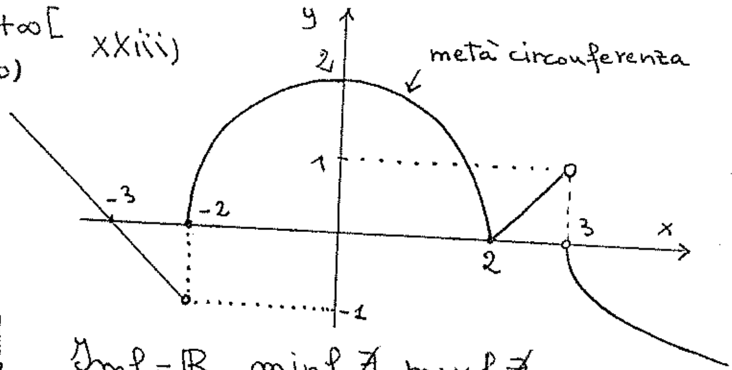
$\min f = -1 = f(0)$

$\max f \nexists$

$f \searrow$ su $]-\infty, 0]$

$\nearrow$ su $[0, +\infty[$

xxiii)

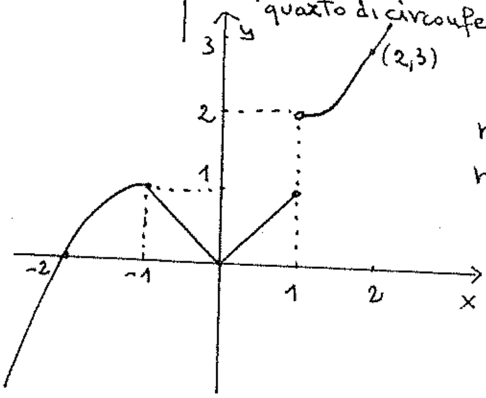


$\Im m f = \mathbb{R}$ $\min f \nexists$ $\max f \nexists$

$f \nearrow [-2,0]$, $[2,3[$

$\searrow]-\infty,-2[$, $[0,2]$, $]3,+\infty[$

xxiv)



$\min f \nexists$

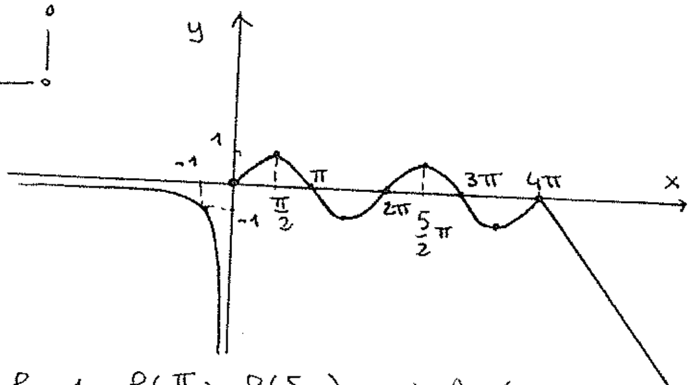
$\max f \nexists$

$\Im m f =]-\infty, 1] \cup]2, +\infty[$

$f \nearrow]-\infty,-1]$, $[0,+\infty[$

$\searrow [-1,0]$

xxv)



$\Im m f =]-\infty, 1]$ $\max\limits_{\mathbb{R}} f = 1 = f(\frac{\pi}{2}) = f(\frac{5}{2}\pi)$ $\min f \nexists$

$f \searrow]-\infty,0[$, $[\frac{\pi}{2}, \frac{3}{2}\pi]$, $[\frac{5}{2}\pi, \frac{7}{2}\pi]$, $[4\pi, +\infty[$ $\boxed{-1 \leq f(x) < 0}$ $x \in]-\infty,-1] \cup]\pi, 2\pi[\cup$

$\nearrow [0, \frac{\pi}{2}]$, $[\frac{3}{2}\pi, \frac{5}{2}\pi]$ $[\frac{7}{2}\pi, 4\pi]$ $\cup]3\pi, 4\pi[\cup]4\pi, 4\pi+1]$

166) i) $]-\infty,-\sqrt{3}]\cup[\sqrt{3},+\infty[$ ii) $\bigcup_{k\in\mathbb{Z}}]\frac{\pi}{4}+2k\pi,\frac{5}{4}\pi+2k\pi[$

iii) $]-\infty,\frac{1}{3}[\cup]1,+\infty[$ iv) $]-\infty,-1]\cup]0,+\infty[$

v) $\mathbb{R}\setminus\{0,1\}$ vi) $\mathbb{R}$ vii) $\mathbb{R}\setminus\{2\}$

166 bis) $\mathrm{dom} f =]1,+\infty[$ $\mathrm{dom}\, g =]-\infty,0]\cup]1,+\infty[$

le due funzioni sono diverse in quanto non hanno lo stesso dominio, dove sono entrambe definite coincidono.

167) a) $\mathrm{dom} f \begin{cases} x^2-5x+6>0 & x<2 \cup x>3 \\ 4x+3\geq 0 & x\geq -\frac{3}{4} \end{cases}$ $\mathrm{dom} f = [-\frac{3}{4},2[\cup]3,+\infty[$

b) $\mathrm{dom} f: x^2-1\geq 0$ $x\leq -1\cup x\geq 1$ $\mathrm{dom} f =]-\infty,-1]\cup[1,+\infty[$

c) $\mathrm{dom} f \begin{cases} 9-x^2\geq 0 & -3\leq x\leq 3 \\ 1-2x>0 & x<\frac{1}{2} \\ e^{2x}\neq 0 & \forall x \end{cases}$ $\mathrm{dom} f = [-3,\frac{1}{2}[$

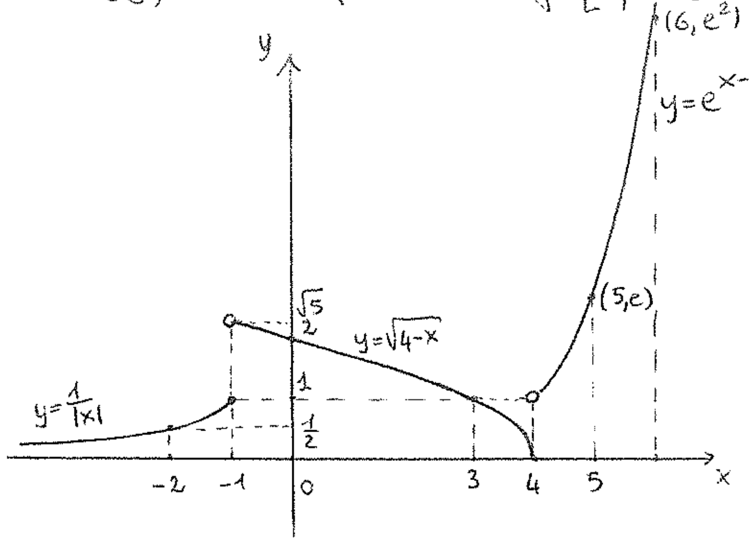
d) $\mathrm{dom} f \begin{cases} 9x-4x^2-2\neq 0 & x\neq \frac{1}{4},2 \\ x>0 \end{cases}$ $\mathrm{dom} f =]0,\frac{1}{4}[\cup]\frac{1}{4},2[\cup]2,+\infty[$

i) $\mathrm{dom} f \begin{cases} 6-x^2+x\geq 0 & -2\leq x\leq 3 \\ 2-3x\neq 0 & x\neq\frac{2}{3} \end{cases}$ $\mathrm{dom} f = [-2,\frac{2}{3}[\cup]\frac{2}{3},3]$

e) $\mathrm{dom} f \begin{cases} 1-\frac{3}{2}x-x^2>0 & -2<x<\frac{1}{2} \\ 16x^2-1\neq 0 & x\neq\pm\frac{1}{4} \\ 5x+7\geq 0 & x\geq -7/5 \end{cases}$ $\mathrm{dom} f = [-\frac{7}{5},-\frac{1}{4}[\cup]-\frac{1}{4},\frac{1}{4}[\cup]\frac{1}{4},\frac{1}{2}[$

f) $\mathrm{dom} f \begin{cases} (x+2)(x+1)-2>0 & x^2+3x>0 & x<-3\cup x>0 \\ x^2+10x+25\neq 0 & (x+5)^2\neq 0 & x\neq -5 \end{cases}$ $\mathrm{dom} f =]-\infty,-5[\cup]-5,-3[\cup]0,+\infty[$

168) a) $\text{dom} f = \mathbb{R}$ $\quad \text{Im} f = [0, +\infty[$



$y = \frac{1}{|x|} = -\frac{1}{x}$ $\quad x \leq -1$

$y = \sqrt{4-x} = \sqrt{-(x-4)}$ cioè $y = \sqrt{-x}$

a destra di 4

$y = e^{x-4}$ $\quad y = e^x$ a destra di 4

$y(-1) = 1$ $\quad y(-2) = \frac{1}{2}$

$y(0) = 2$ $\quad y(3) = 1$ $\quad y(4) = 0$

$y(5) = e$ $\quad y(6) = e^2$

$y = \sqrt{4-x}$ in $x = -1$ dà $\sqrt{5} \simeq 2{,}24$

$y = e^{x-4}$ in $x = 4$ dà 1

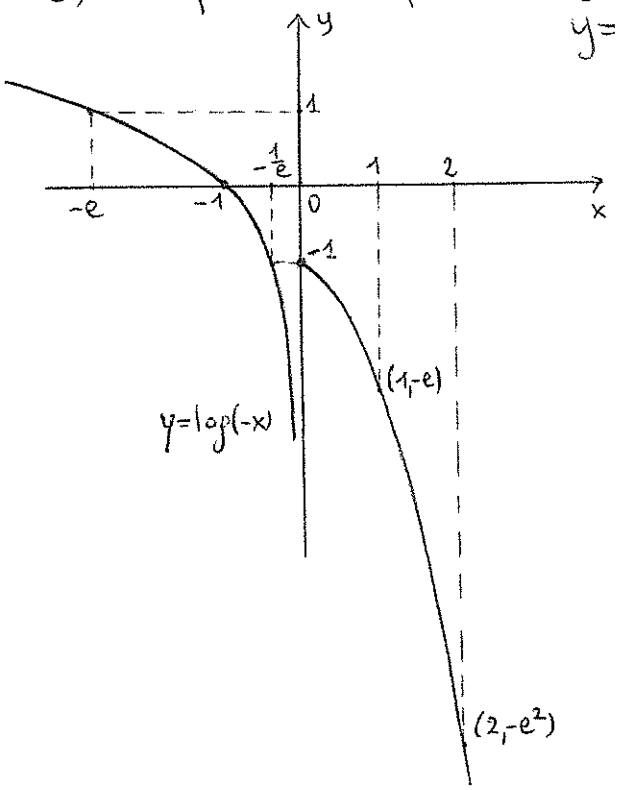
$f(x) = k$ $\quad k < 0$ ϕ $\quad 0 < k < \sqrt{5}$ 2 sol.ni $\quad k \geq \sqrt{5}$ 1 sol.ne

$f \nearrow$ $]-\infty, -1]$ e su $[4, +\infty[$

$f \searrow$ $]-1, 4]$

$\boxed{f(x) \geq 1}$ $\quad x \in [-1, 3] \cup]4, +\infty[$

b) $\text{dom} f = \mathbb{R}$ $\quad \text{Im} f = \mathbb{R}$



$y = \log(-x)$ simmetrico di $y = \log x$ risp asse y

$y = -e^x$ simmetrico di $y = e^x$ risp asse x

$f(-e) = 1$ $\quad f(-1) = 0$ $\quad y(-\frac{1}{e}) = -1$

$f(0) = -1$ $\quad f(1) = -e$ $\quad f(2) = -e^2$

$f(x) = k$ $\quad k \leq -1$ 2 sol.ni $\quad k > -1$ 1 sol.ne

$f \searrow$ su $]-\infty, 0[$ e su $[0, +\infty[$

c) $\text{dom} f = \mathbb{R} \setminus \{2\}$ $\quad \text{Im} f = \mathbb{R}$

$y = 1 + e^{-x-1} = 1 + e^{-(x+1)}$ è $y = e^{-x}$ spostato a sin di 1 e in alto di 1

$y = \frac{1}{(x-2)^2}$ è $y = \frac{1}{x^2}$ a destra di 2

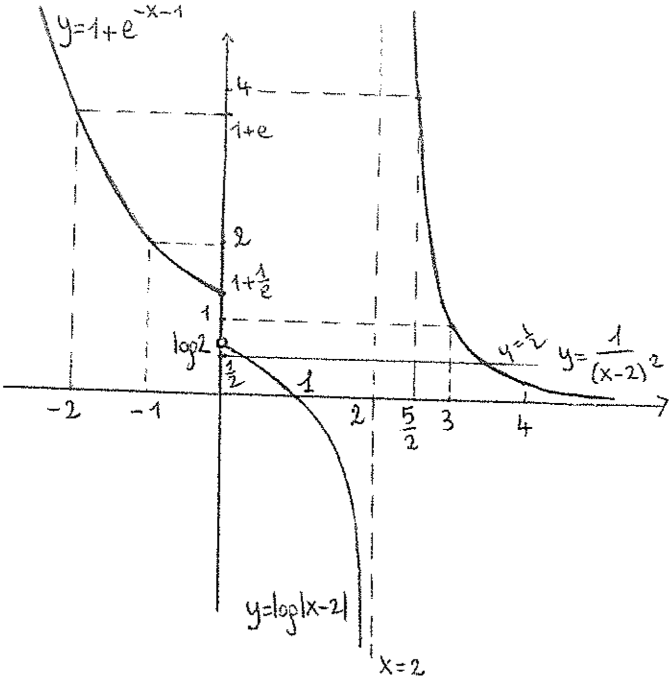
$y = \log|x-2|$ è $y = \log|x|$ spostato a destra di 2

$y(-2) = 1 + e$ $\quad y(-1) = 2$ $\quad y(0) = 1 + \frac{1}{e}$

$y(1) = 0$ $\quad y = \log|x-2|$ in $x = 0$ $\rightarrow \log 2 \simeq 0{,}69$

$y(3) = 1$ $\quad y(4) = \frac{1}{4}$ $\quad y(\frac{5}{2}) = 4$

c)



$f(x)=K \quad K\le 0 \quad 1$ sol.ne

$0<K<\log 2 \quad 2$ sol.ni

$\log 2<K<1+\frac{1}{e} \quad 1$ sol.ne

$K\ge 1+\frac{1}{e} \quad 2$ sol.ni

$f \downarrow \ ]-\infty,2[$ e su $]2,+\infty[$

$\boxed{f(x)=\frac{1}{2}}$ $\quad \log|x-2|=\frac{1}{2}=\log e^{1/2}$

$0<x<2$

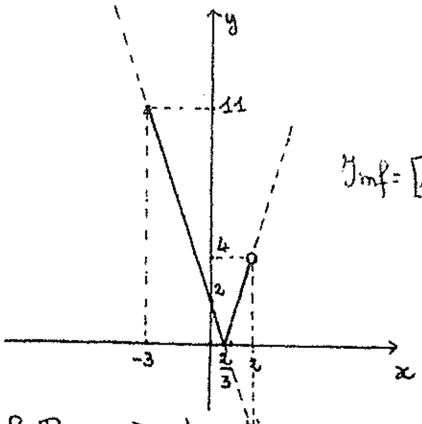
$|x-2|=\sqrt{e} \quad x-2=\pm\sqrt{e} \quad x=2\pm\sqrt{e}$

$\boxed{x=2-\sqrt{e}}$

$\frac{1}{(x-2)^2}=\frac{1}{2} \quad (x-2)^2=2 \quad x-2=\pm\sqrt{2}$

$x>2 \quad x=2\pm\sqrt{2} \quad \boxed{x=2+\sqrt{2}}$

d)



$\mathrm{Im} f=[0,11]$

dom f $=\mathbb{R}$ ma è da disegnare su $[-3,2[$

$y=2-3x$ retta per $(0,2)$ $(\frac{2}{3},0)$

$(-3,11)$

$(2,-4)$

$y=|2-3x|$ si deve ribaltare la parte al di sotto dell'asse x $\quad f(0)=2,\ f(-3)=11$

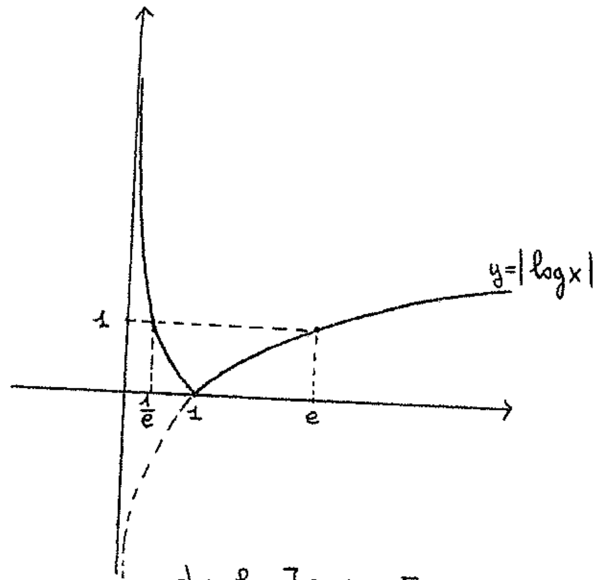
$f(\frac{2}{3})=0 \quad f(2)=4$

$f(x)=K \quad K<0$ o $K>11$ nessuna sol.ne

$K=0$ o $4\le K\le 11 \quad 1$ sol.ne

$0<K<4 \quad 2$ sol.ni

e)

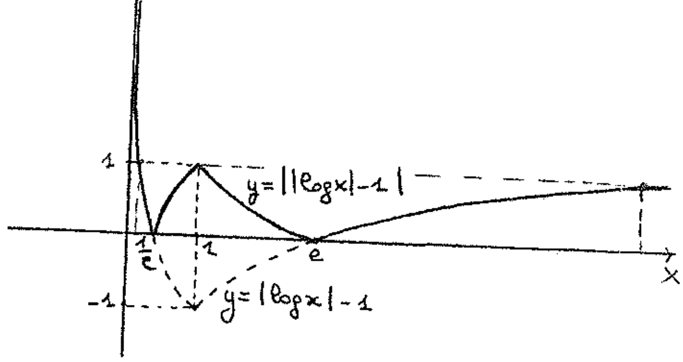


dom f $=]0,+\infty[$

$|\log x|=1 \iff \log x=1$ o $\log x=-1$

$\iff x=e$ o $x=\frac{1}{e}$

$\mathrm{Im} f=[0,+\infty[$



$\boxed{f(x)=1}$ $\quad ||\log x|-1|=1 \iff |\log x|-1=1$ o $|\log x|-1=-1 \iff |\log x|=2$ o $|\log x|=0 \iff$

$\iff \log x=\pm 2$ o $\log x=0 \iff \boxed{x=\frac{1}{e^2},\ x=e^2,\ x=1}$

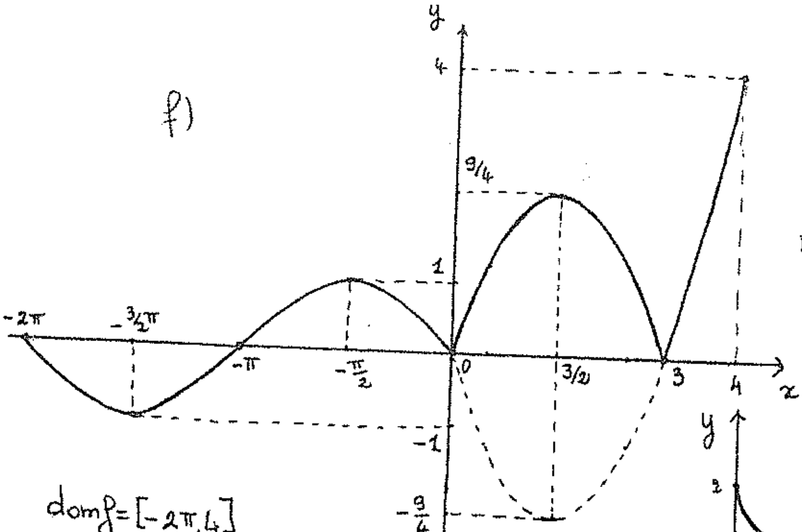
$f(x)=K \quad K<0$ nessuna sol.ne

$K=0$ o $K>1 \quad 2$ sol.ni

$K=1 \quad 3$ sol.ni

$0<K<1 \quad 4$ sol.ni

f)



$\mathrm{Im} f = [-1, 4]$

min loc : $x = -\frac{3}{2}\pi$, $x = 0$, $x = 3$

max loc : $x = -2\pi$, $x = -\frac{\pi}{2}$, $x = \frac{3}{2}$, $x = 4$

$\mathrm{dom} f = [-2\pi, 4]$

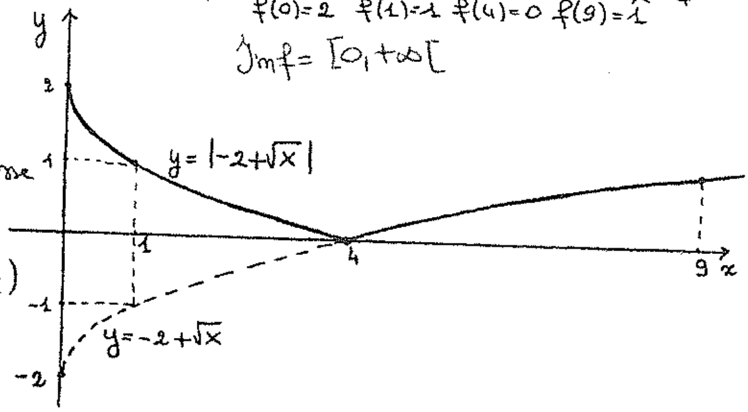
$y = \mathrm{sen}|x|$ simmetrico di $y = \mathrm{sen}\,x$ rispetto all'asse delle y

$y = x^2 - 3x$ parabola verso l'alto $V\left(\frac{3}{2}, -\frac{9}{4}\right)$

g) $\mathrm{dom} f = [0, +\infty[$ $\quad \sqrt{x} - 2 = 0 \quad \sqrt{x} = 2 \quad x = 4$

$f(0) = 2 \quad f(1) = 1 \quad f(4) = 0 \quad f(9) = 1$

$\mathrm{Im} f = [0, +\infty[$



$K < -1$ o $K > 4$ nessuna sol.^{ne}

$K = -1$ o $\frac{9}{4} < K \leq 4$ 1 sol.^{ne}

$-1 < K < 0$ 2 sol.^{ni} $K = 9/4$

$1 < K < 9/4$ 3 sol.^{ni}

$K = 0$, $K = 1$ 4 sol.^{ni}

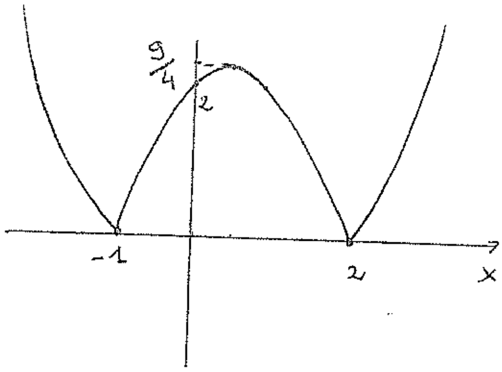
$0 < K < 1$ 5 sol.^{ni}

$K < 0$ nessuna sol.^{ne}

$K = 0$ o $K > 2$ 1 sol.^{ne}

$0 < K \leq 2$ 2 sol.^{ni}

h)



$\mathrm{Im} f = [0, +\infty[$ $\quad \mathrm{dom} f = \mathbb{R}$

$f(x) = K$ ha 0 sol.^{ni} $K < 0$

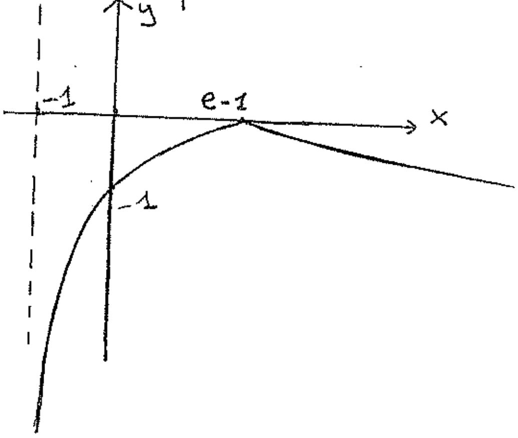
2 sol.^{ni} $K = 0$

4 sol.^{ni} $0 < K < \frac{9}{4}$

3 sol.^{ni} $K = 9/4$

2 sol.^{ni} $K > 9/4$

i) $\mathrm{dom} f =]-1, +\infty[$ $\qquad \mathrm{Im} f =]-\infty, 0]$

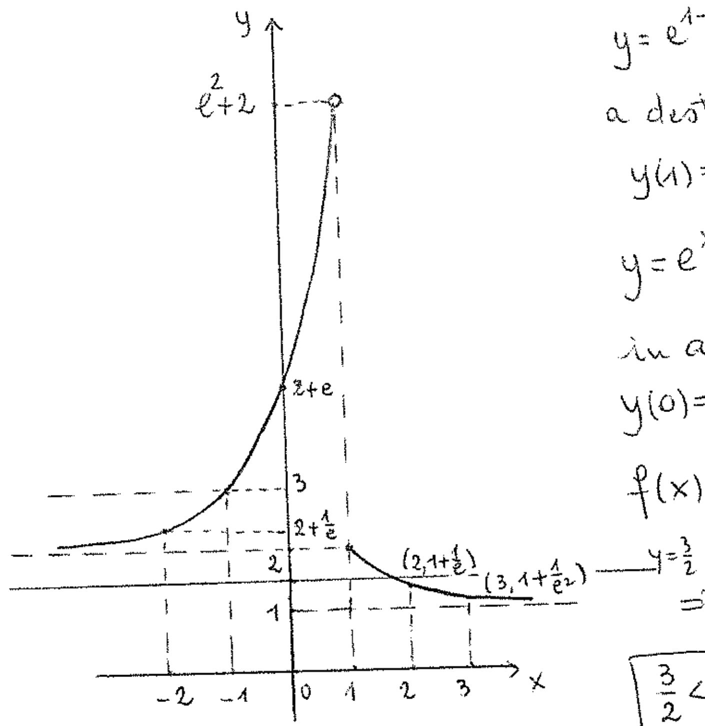


$f(x) = K$ ha 2 sol.^{ni} $K < 0$

1 sol.^{ne} $K = 0$

0 sol.^{ni} $K > 0$

k) $\text{dom} f = \mathbb{R}$ $\text{Im} f =]1, e^2+2[$



$y = e^{1-x} + 1 = e^{-(x-1)} + 1$ è $y = e^{-x}$ a destra di 1 e in alto di 1

$y(1) = 2$ $y(2) = 1 + \frac{1}{e}$ $y(3) = 1 + \frac{1}{e^2}$

$y = e^{x+1} + 2$ è $y = e^x$ a sin di 1 e in alto di 2 in $x=1$ sarebbe $y = e^2+2$

$y(0) = 2+e$ $y(-1) = 3$ $y(-2) = 2 + \frac{1}{e}$

$f(x) = k$ $k \leq 1$ o $k \geq e^2+2$ nessuna sol.ne

$k \in]1, e^2+2[$ 1 sol.ne

$\Rightarrow f$ è INIETTIVA

$\boxed{\frac{3}{2} < f(x) < 3}$ $e^{1-x} + 1 = \frac{3}{2}$ $e^{1-x} = \frac{1}{2}$

$e^{1-x} = e^{\log\frac{1}{2}} = e^{-\log 2} \iff 1-x = -\log 2$ $x = 1 + \log 2$

e^x biunivoca

$\boxed{x \in]-\infty, -1[\cup [1, 1+\log 2[}$

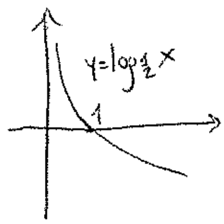
169) a) $\text{dom} f = \{x \in \mathbb{R} : 1-2x > 0, \log(1-2x) - 1 \geq 0\}$

$\begin{cases} 1-2x > 0 \\ \log(1-2x) \geq 1 = \log e \end{cases}$ $\begin{cases} x < \frac{1}{2} \\ 1-2x \geq e \end{cases}$ $\begin{cases} x < \frac{1}{2} \\ x \leq \frac{1-e}{2} \end{cases}$ $x \in]-\infty, \frac{1-e}{2}]$

b) $\text{dom} f = \{x \in \mathbb{R} : x > 0, \log_3 x - 2 \neq 0\}$ $\begin{cases} x > 0 \\ \log_3 x \neq \log_3 9 \end{cases}$ $\begin{cases} x > 0 \\ x \neq 9 \end{cases}$

$x \in]0, 9[\cup]9, +\infty[$

c) $\text{dom} f = \{x \in \mathbb{R} : x-1 > 0, \log_{\frac{1}{2}}(x-1) \geq 0\}$ $\begin{cases} x > 1 \\ 0 < x-1 \leq 1 \end{cases}$



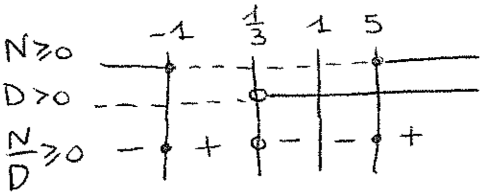
$\begin{cases} x > 1 \\ 1 < x \leq 2 \end{cases}$ $x \in]1, 2]$

d) $\text{dom} f = \{x \in \mathbb{R} ; x > 0, 1 - \log x > 0\}$ $\begin{cases} x > 0 \\ \log x < 1 \end{cases}$ $x \in]0, e[$

e) $\dfrac{|x-2|-3}{|x|+2x-1} \geq 0$

$|x-2| \geq 3 \iff x-2 \leq -3$ o $x-2 \geq 3 \iff x \leq -1$ o $x \geq 5$

$|x| + 2x - 1 > 0 \iff |x| > 1-2x \iff x < 2x-1$ o $x > 1-2x$
$\iff x > 1$ o $x > \frac{1}{3} \iff x > \frac{1}{3}$



$\text{dom} f = [-1, \frac{1}{3}[\cup [5, +\infty[$