

Corso: ELEMENTI di MATEMATICA

SCHEDA di ESERCIZI N°4

TRIGONOMETRIA

1ª parte

1) Convertite da gradi a radianti:

$$150 \text{ gradi} = \frac{5}{6} \pi \text{ radianti}; \quad 15 \text{ gradi} = \frac{\pi}{12} \text{ radianti}; \quad -90 \text{ gradi} = -\frac{\pi}{2} \text{ radianti}$$

$$285 \text{ gradi} = \frac{19}{12} \pi \text{ radianti}; \quad 390 \text{ gradi} = \frac{13}{6} \pi \text{ radianti}; \quad -120 \text{ gradi} = -\frac{2}{3} \pi \text{ radianti}$$

2) Convertite da radianti a gradi:

$$\pi \text{ radianti} = 180 \text{ gradi}; \quad \frac{4\pi}{3} \text{ radianti} = 240 \text{ gradi}; \quad \frac{11\pi}{6} \text{ radianti} = 330 \text{ gradi}$$

$$\frac{\pi}{12} \text{ radianti} = 15 \text{ gradi}; \quad 5\pi \text{ radianti} = 900 \text{ gradi}; \quad -\frac{3\pi}{4} \text{ radianti} = -135 \text{ gradi}$$

D'ora in avanti tutti gli angoli sono espressi in radianti

3)

$$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin \frac{7\pi}{6} = -\frac{1}{2} \quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \sin(3\pi) = 0 \quad \sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos \frac{-2\pi}{3} = -\frac{1}{2} \quad \tan \frac{3\pi}{4} = -1 \quad \tan \frac{\pi}{3} = \frac{\sqrt{3}}{3} \quad \cos \frac{5\pi}{2} = 0 \quad \cos(-\pi) = -1$$

Esercizi nella dispensa caricata su ELLY da pag. 89 a pag. 91 (es. 104)

2ª parte

4) Scrivete quali sono gli angoli $x \in [0, 2\pi]$ che verificano: sol. a pag. 2-3

$$\sin x = \frac{1}{2} \quad \text{Risposta: } x = \frac{\pi}{6}, x = \frac{5}{6} \pi \quad \sin x = -1 \quad \text{Risposta: } x = \frac{3}{2} \pi$$

$$\cos x = \frac{\sqrt{2}}{2} \quad \text{Risposta: } x = \frac{\pi}{4}, x = \frac{7}{4} \pi \quad \cos x = -\frac{\sqrt{3}}{2} \quad \text{Risposta: } x = \frac{5}{6} \pi, x = \frac{7}{6} \pi$$

$$\tan x = -1 \quad \text{Risposta: } x = \frac{3}{4} \pi, x = \frac{7}{4} \pi \quad \tan x = \sqrt{3} \quad \text{Risposta: } x = \frac{\pi}{3}, x = \frac{4}{3} \pi$$

$$(2 \sin x - \sqrt{3}) \cdot (\sqrt{2} \cos x + 1) = 0 \quad \text{Risposta } \dots x = \frac{\pi}{3} \quad x = \frac{2}{3} \pi \quad x = \frac{3}{4} \pi \quad x = \frac{5}{4} \pi$$

$$2 \cos^3 x + 3 \cos^2 x + \cos x = 0 \quad \text{Risposta } \dots x = \frac{\pi}{2} \quad x = \frac{2}{3} \pi \quad x = \pi \quad x = \frac{4}{3} \pi \quad x = \frac{3}{2} \pi$$

$$2 \cos^2 x - \sin x - 2 = 0 \quad \text{Risposta } \dots x = 0 \quad x = \pi \quad x = \frac{7}{6} \pi \quad x = \frac{11}{6} \pi \quad x = 2\pi$$

$$\frac{-2 \cos x + \sqrt{3}}{5} = 0 \quad \text{Risposta } \dots x = \frac{\pi}{6} \quad x = \frac{11}{6} \pi$$

$$2 \sin^2 x + \sin x - 1 = 0 \quad \text{Risposta } \dots x = \frac{\pi}{6} \quad x = \frac{5}{6} \pi \quad x = \frac{3}{2} \pi$$

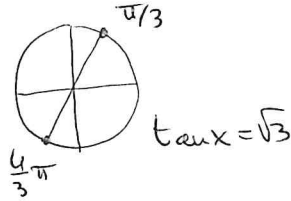
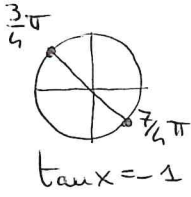
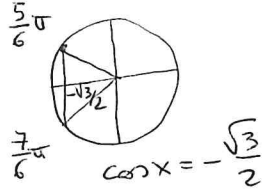
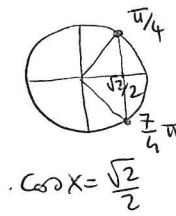
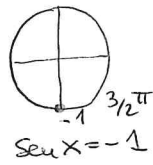
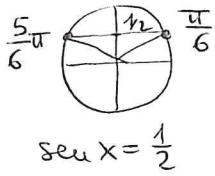
$$\frac{\sqrt{3} - 3 \tan x}{1 + \cos^2 x} = 0 \quad \text{Risposta } \dots x = \frac{\pi}{6} \quad x = \frac{7}{6} \pi \quad -8 \sin x - 16 \sin x \cos x = 0$$

$$\text{Risposta } x = 0 \quad x = \frac{2}{3} \pi \quad x = \pi \quad x = \frac{4}{3} \pi \quad x = 2\pi$$

Esercizi su ELLY da 105) (pag. 91, solo $x \in [0, 2\pi]$) a 111)

+ es. 112) pag. 93 (Programma dei soli Matematici).

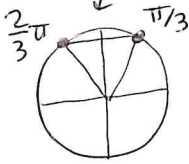
es. 4)



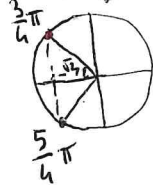
$(2 \sin x - \sqrt{3}) \cdot (\sqrt{2} \cos x + 1) = 0 \iff 2 \sin x - \sqrt{3} = 0 \quad \vee \quad \sqrt{2} \cos x + 1 = 0$

legge di annullamento del prodotto

$\iff \sin x = \frac{\sqrt{3}}{2}$

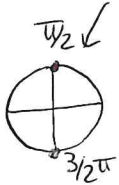


$\iff \cos x = -\frac{\sqrt{2}}{2}$



$2 \cos^3 x + 3 \cos^2 x + \cos x = 0 \iff \cos x \cdot (2 \cos^2 x + 3 \cos x + 1) = 0$

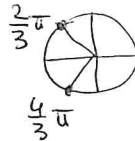
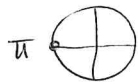
$\iff \cos x = 0 \quad \vee \quad 2 \cos^2 x + 3 \cos x + 1 = 0$



$t = \cos x \quad 2t^2 + 3t + 1 = 0$

$(t+1)(2t+1) = 0 \quad t_1 = -1 \quad t_2 = -\frac{1}{2}$

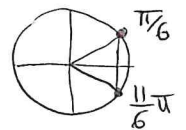
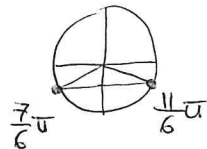
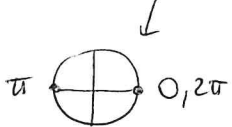
$\cos x = -1 \quad \vee \quad \cos x = -\frac{1}{2}$



$2 \cos^2 x - \sin x - 2 = 0 \iff 2(1 - \sin^2 x) - \sin x - 2 = 0 \iff \cos^2 x = 1 - \sin^2 x$

$\iff -2 \sin^2 x - \sin x = 0 \iff \sin x (2 \sin x + 1) = 0$

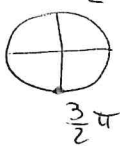
$\iff \sin x = 0 \quad \vee \quad 2 \sin x + 1 = 0$

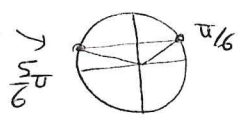


$\frac{-2 \cos x + \sqrt{3}}{5} = 0 \iff -2 \cos x + \sqrt{3} = 0 \iff \cos x = \frac{\sqrt{3}}{2}$

$$2 \sec^2 x + \sec x - 1 = 0 \quad \sec x = t \quad 2t^2 + t - 1 = 0$$

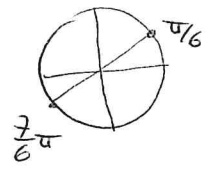
$$(t+1)(2t-1) = 0 \quad \sec x = -1 \quad \text{or} \quad \sec x = \frac{1}{2}$$





$$\frac{\sqrt{3} - 3 \tan x}{1 + \cos^2 x} = 0 \Leftrightarrow \sqrt{3} - 3 \tan x = 0 \Leftrightarrow \tan x = \frac{\sqrt{3}}{3}$$

poiché $1 + \cos^2 x \geq 1 \quad \forall x$
 e quindi $\neq 0 \quad \forall x$



C.E. della $\tan x$
 $x \neq \frac{\pi}{2} \quad x \neq \frac{3}{2}\pi$

$$-8 \sec x \cdot (1 + 2 \cos x) = 0 \Leftrightarrow \sec x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}$$

