

DISEQUAZIONI ESPONENZIALI E LOGARITMICHE

1) Risolvete le seguenti disequazioni esponenziali:

$$4^x \leq \frac{1}{16} \quad 5^x \geq 125 \quad 3^{\frac{7}{4}x-1} < 3^{\frac{1}{2}x+\frac{2}{3}} \quad 10^{9x^2+12x} > \frac{1}{10'000}$$

$$2^{4x^2-1} \leq 1 \quad e^{3x} > -1 \quad 3^{x^2} \leq -3^2 \quad e^{9x^2} \cdot e^{6x} > \frac{1}{e}$$

$$2^{-\frac{x}{3}} \leq 128 \quad e^{5x-2} > e^{4+x}$$

2) Risolvete le seguenti disequazioni logaritmiche:

$$\log_3 x > -2 \quad \log_{10} x < \frac{1}{2} \quad \log_x < 1 \quad \log(3x^2) > 0$$

$$\log(3-7x) > -1 \quad \log_4(3x-1) < 2 \quad \log(4x^2-1) < \log(-1-x)$$

$$\log(4x^2-1) > \log(-1-x) \quad \log_{10}(10-9x) < 2 \quad \log_4 2x > \frac{1}{2}$$

$$\log(2x^2+5) \geq \log(-5x)$$

3) Semplificate specificando per quali valori di x vale l'uguaglianza:

glianza:

$$e^{\log(3x-2)} = \dots$$

$$\log e^{x^2-4} = \dots$$

$$e^{\log 2 - \log 3 + x} = \dots$$

$$e^{\log(x^2-2) + \log(1-x)} = \dots$$

$$e^{\log\left(\frac{x^2-25}{2-3x}\right)} = \dots$$

4) Scrivete come esponenziali di base il numero e :

$$2^5 = \dots$$

$$3^x = \dots$$

$$4^{2-3x} = \dots$$

→ Esercizi sulle DISPENSE su ELLY es. 156 - 158 - 159 (iii) pag. 141-2

Sol. me Dis^w ESP-LOG -1-

$$\text{es. 1) } 4^x \leq 4^{-2} \xrightarrow{4^x \uparrow} x \leq -2 \quad 3^{\frac{7}{4}x-1} < 3^{\frac{1}{2}x+\frac{2}{3}} \xrightarrow{3^x \uparrow} \frac{7}{4}x-1 < \frac{1}{2}x+\frac{2}{3}$$

$$\xrightarrow{3^x \uparrow} \frac{5}{4}x < \frac{5}{3} \Leftrightarrow x < \frac{4}{3}$$

$$10^{9x^2+12x} > 10^{-4} \xrightarrow{10^x \uparrow} 9x^2+12x > -4 \Leftrightarrow 9x^2+12x+4 > 0$$

$$\Leftrightarrow (3x+2)^2 > 0 \Leftrightarrow \forall x \neq -\frac{2}{3}$$

$$2^{4x^2-1} \leq 2^0 \xrightarrow{2^x \uparrow} 4x^2-1 \leq 0 \Leftrightarrow x^2 \leq \frac{1}{4} \Leftrightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$e^{3x} > -1 \quad \forall x \quad 3^{x^2} \leq -3^2 \Leftrightarrow 3^{x^2} \leq -9 \text{ IMPOSSIBLE}$$

$$e^{9x^2+6x} > e^{-1} \xrightarrow{e^x \uparrow} 9x^2+6x > -1 \Leftrightarrow 9x^2+6x+1 > 0$$

$$\Leftrightarrow (3x+1)^2 > 0 \Leftrightarrow \forall x \neq -\frac{1}{3}$$

$$2^{-\frac{x}{3}} \leq 2^7 \xrightarrow{2^x \uparrow} -\frac{x}{3} \leq 7 \Leftrightarrow x \geq -21$$

$$e^{5x-2} > e^{4+x} \xrightarrow{e^x \uparrow} 5x-2 > 4+x \Leftrightarrow 4x > 6 \Leftrightarrow x > \frac{3}{2}$$

$$\text{es. 2) } \log_3 x > -2 \Leftrightarrow \begin{cases} x > 0 \text{ c.e.} \\ \log_3 x > \log_3 \frac{1}{9} \end{cases} \xrightarrow{\log_3 x \uparrow} \begin{cases} x > 0 \\ x > \frac{1}{9} \end{cases} \Leftrightarrow x > \frac{1}{9}$$

$$\log_{10} x < \frac{1}{2} \Leftrightarrow \begin{cases} x > 0 \text{ c.e.} \\ \log_{10} x < \log_{10} \sqrt{10} \end{cases} \xrightarrow{\log_{10} x \uparrow} \begin{cases} x > 0 \\ x < \sqrt{10} \end{cases} \Leftrightarrow 0 < x < \sqrt{10}$$

$$\log x < 1 \Leftrightarrow \begin{cases} x > 0 \text{ c.e.} \\ \log x < \log e \end{cases} \xrightarrow{\log x \uparrow} \begin{cases} x > 0 \\ x < e \end{cases} \Leftrightarrow 0 < x < e$$

$$\log(3x^2) > 0 \Leftrightarrow \begin{cases} 3x^2 > 0 \text{ c.e.} \\ \log(3x^2) > \log 1 \end{cases} \xrightarrow{\log x \uparrow} \begin{cases} x \neq 0 \\ 3x^2 > 1 \end{cases} \Leftrightarrow \begin{cases} x \neq 0 \\ x^2 > \frac{1}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} x \neq 0 \\ x < -\frac{\sqrt{3}}{3} \text{ o } x > \frac{\sqrt{3}}{3} \end{cases} \Leftrightarrow x < -\frac{\sqrt{3}}{3} \text{ o } x > \frac{\sqrt{3}}{3}$$

$$\log(3-7x) > -1 \Leftrightarrow \begin{cases} 3-7x > 0 \text{ c.e.} \\ \log(3-7x) > \log \frac{1}{e} \end{cases} \xrightarrow{\log x \uparrow} \begin{cases} x < \frac{3}{7} \\ 3-7x > \frac{1}{e} \end{cases} \Leftrightarrow \begin{cases} x < \frac{3}{7} \\ 7x < 3-\frac{1}{e} \end{cases}$$

$$\Leftrightarrow \begin{cases} x < \frac{3}{7} \\ x < \frac{1}{7}(3-\frac{1}{e}) \end{cases} \Leftrightarrow x < \frac{1}{7}(3-\frac{1}{e})$$

SOL. ⁿⁱ DIS. ⁿⁱ ESP-LOG-2-

$$\log_4(3x-1) < 2 \Leftrightarrow \begin{cases} 3x-1 > 0 \\ \log_4(3x-1) < \log_4 16 \end{cases} \begin{matrix} \text{D} > 0 \\ \log_4 x \uparrow \end{matrix} \left\{ \begin{array}{l} x > \frac{1}{3} \\ 3x-1 < 16 \end{array} \right. \Leftrightarrow \begin{cases} x > \frac{1}{3} \\ x < \frac{17}{3} \end{cases}$$

$$\Leftrightarrow \frac{1}{3} < x < \frac{17}{3}$$

$$\log(4x^2-1) < \log(-1-x) \Leftrightarrow \begin{cases} 4x^2-1 > 0 \text{ c.e.} \\ -1-x > 0 \text{ c.e.} \\ \log x \uparrow \end{cases} \Leftrightarrow \begin{cases} 4x^2-1 < -1-x \\ x < -\frac{1}{2} \text{ o } x > \frac{1}{2} \\ x < -1 \\ 4x^2+x < 0 \\ x(4x+1) < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 1^a \cap 2^a & x < -1 \\ -\frac{1}{4} < x < 0 \end{cases} \quad \phi \text{ nessuna sol. } \text{ve}$$

$$\log(4x^2-1) > \log(-1-x) \Leftrightarrow \begin{cases} 1^a \cap 2^a & x < -1 \text{ c.e.} \\ 4x^2-1 > -1-x \\ 4x^2+x > 0 \end{cases} \Leftrightarrow \begin{cases} x < -1 \\ x < -\frac{1}{4} \text{ o } x > 0 \end{cases}$$

$$\Leftrightarrow \boxed{x < -1}$$

$$\log_{10}(10-9x) < 2 \Leftrightarrow \begin{cases} 10-9x > 0 \\ \log_{10}(10-9x) < \log_{10} 100 \end{cases} \begin{matrix} \text{D} > 0 \\ \log_{10} x \uparrow \end{matrix} \left\{ \begin{array}{l} x < \frac{10}{9} \\ 10-9x < 100 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x < \frac{10}{9} \\ x > -10 \end{cases} \Leftrightarrow -10 < x < \frac{10}{9}$$

$$\log_4 2x > \frac{1}{2} \Leftrightarrow \begin{cases} 2x > 0 \\ \log_4 2x > \log_4 \sqrt{4} \end{cases} \begin{matrix} \text{D} > 0 \\ \log_4 x \uparrow \end{matrix} \left\{ \begin{array}{l} x > 0 \\ 2x > 2 \end{array} \right. \Leftrightarrow \begin{cases} x > 0 \\ x > 1 \end{cases} \Leftrightarrow x > 1$$

$$\log(2x^2+5) > \log(-5x) \Leftrightarrow \begin{cases} 2x^2+5 > 0 \text{ c.e.} \\ -5x > 0 \text{ c.e.} \\ 2x^2+5 > -5x \text{ (log)} \end{cases} \Leftrightarrow \begin{cases} \forall x \\ x < 0 \\ 2x^2+5x+5 \geq 0 \end{cases}$$

$$\Delta < 0$$

$$x_{1,2} = \frac{-5 \pm \sqrt{-15}}{4}$$

$$\forall x$$

$$\Leftrightarrow \begin{cases} \forall x \\ x < 0 \\ \forall x \end{cases} \Leftrightarrow x < 0$$

ES.3) $e^{\log(3x-2)} = 3x-2 \quad \forall x: 3x-2 > 0 \text{ cioè } \forall x > \frac{2}{3}$

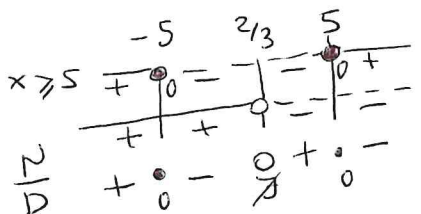
$$\log e^{x^2-4} = x^2-4 \quad \forall x$$

$$e^{\log 2 - \log 3 + x} = e^{\log \frac{2}{3} + x} = \frac{2}{3} e^x \quad \forall x$$

$$e^{\log(x^2-2) + \log(1-x)} = e^{\log((x^2-2)(1-x))} = (x^2-2)(1-x) = -x^3 + x^2 + 2x - 2$$

$$\forall x: \begin{cases} x^2-2 > 0 \\ 1-x > 0 \end{cases} \left\{ \begin{array}{l} x < -\sqrt{2} \text{ o } x > \sqrt{2} \\ x < 1 \end{array} \right. \text{ cioè } \forall x < -\sqrt{2}$$

$$e^{\log\left(\frac{x^2-25}{2-3x}\right)} = \frac{x^2-25}{2-3x} \text{ se } \frac{x^2-25}{2-3x} > 0 \quad \begin{matrix} N \geq 0 & x \leq -5 \text{ o } x \geq 5 \\ D > 0 & x < \frac{2}{3} \end{matrix}$$



ES.4) $2^5 = e^{5 \log 2}$ $3^x = e^{x \log 3}$ $4 = e^{(2-3x) \log 4}$
 $\hookrightarrow 2^5 = e^{\log 2^5} = e^{5 \log 2}$