EX 1

TROVARE LE SOLUZIONI $x \in \mathbb{R}$ DI

$$\frac{1}{2} - \frac{1}{3}\left[x - 2(1-3x)\right] < \frac{x-1}{6} - \frac{x+1}{5} \iff$$

$$\frac{1}{2} - \frac{1}{3}\left[\underbrace{x - 2 + 6x}_{7x-2}\right] - \frac{x-1}{6} + \frac{x+1}{5} < 0 \iff$$

$$\frac{1}{2} - \frac{7}{3}x + \frac{2}{3} - \frac{x-1}{6} + \frac{x+1}{5} < 0 \iff$$

$$\frac{15 - 70x + 20 - 5x + 5 + 6x + 6}{30} < 0 \iff$$

$$\frac{-69x + 46}{30} < 0 \iff -69x + 46 < 0 \iff$$

$$x > \frac{\cancel{46}^{\,2}}{\cancel{69}_{\,3}} = \frac{2}{3}$$

SOLUZIONE $x \in \left]\frac{2}{3}, +\infty\right[$

Ex 2

TROVARE LE SOLUZIONI DI

$$\Big[\underbrace{(x-1)^2-2x+3}_{F_1(x)}\Big]\cdot\Big(\underbrace{3x^2-5x+2}_{F_2(x)}\Big)>0$$

$$F_1(x)\geq 0 \iff x^2-2x+1-2x+3\geq 0 \iff$$

$$\iff x^2-4x+4\geq 0 \iff$$

$$(x-2)^2\geq 0 \quad \forall x\in\mathbb{R}$$

$$=0 \text{ sse } x=2$$

$$F_2(x)\geq 0 \iff 3x^2-5x+2\geq 0 \iff$$

$$\Delta = 25-24=1>0$$

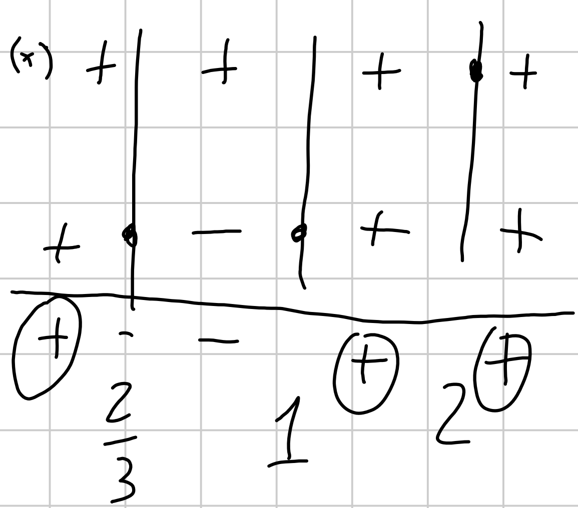
$$3x^2-5x+2=0 \iff x_{1,2}=\frac{5\pm\sqrt{\Delta}}{6}$$

$$x_1=1 \quad x_2=\frac{2}{3}$$

NB $\quad 3x^2-5x+2 = 3(x-1)\left(x-\frac{2}{3}\right)$

$\iff x \le \frac{2}{3} \ \vee\ x \ge 1$

$F_1(x)\cdot F_2(x) > 0 \iff$



SOLUZIONE

$x \in \left]-\infty, \frac{2}{3}\right[\ \cup\ ]1,2[\ \cup$

$\cup\ ]2,+\infty[$

Ex 3

SIANO $\quad A = \left\{ x \in \mathbb{R} \text{ t.c. } \frac{2x+1}{2x-1} \ge \frac{x-2}{2-4x} + 2 \right\}$

$B = \left\{ x \in \mathbb{R} \text{ t.c. } \frac{(x^2-1)(2x+4)}{(1-3x)(2-5x)} \ge 0 \right\}$

DETERMINARE $A,\ B,\ A\cap B,\ A\cup B,\ A\setminus B$

(A) $\dfrac{2x+1}{2x-1} \geq \dfrac{x-2}{2-4x} + 2 \; \star \iff$

C.E

$2x-1 \neq 0$

$2-4x \neq 0$ $\iff x \neq \dfrac{1}{2}$

$\Big\{ \quad \dfrac{x-2}{x-2} > 0 \qquad \mathbb{R} \setminus \{2\} \qquad \{$

$\star \iff \dfrac{2x+1}{2x-1} - \dfrac{x-2}{2(1-2x)} - 2 \geq 0 \iff$

$$\frac{2x+1}{2x-1} + \frac{x-2}{2(2x-1)} - 2 \geq 0 \iff$$

$$\frac{2(2x+1) + x - 2 - 2(2(2x-1))}{2(2x-1)} \geq 0 \iff$$

$$\frac{4x + \cancel{2} + x - \cancel{2} - 8x + 4}{2(2x-1)} \geq 0 \iff$$

$$\frac{-3x+4}{2(2x-1)} \geq 0$$

$$N(x) = -3x + 4 \geq 0 \iff x \leq \frac{4}{3}$$

$$4 \geq 3x$$

$$D(x) = 2(2x-1) > 0 \iff 2x - 1 > 0 \iff x > \frac{1}{2}$$

		$\frac{1}{2}$		$\frac{4}{3}$	
$N(x)$	$+$		$+$	$\bullet$	$-$ $-$
$D(x)$	$-$	0	$+$		$+$
	$-$		$+$		$-$

$$A = \left\{ x \in \mathbb{R} \text{ tc } \frac{1}{2} < x \leq \frac{4}{3} \right\} = \left] \frac{1}{2}, \frac{4}{3} \right]$$

B

$$\frac{(x^2-1)(2x+4)}{(1-3x)(2-5x)} \geq 0$$

CE

$$1 - 3x \neq 0 \iff x \neq \frac{1}{3}$$

$$2 - 5x \neq 0 \iff x \neq \frac{2}{5}$$

$$N_1(x) = x^2 - 1 \geq 0 \iff x \leq -1 \vee x \geq 1$$

$$x^2 - 1 = 0 \iff x = -1 \vee x = 1$$

$N_2(x) = 2x+4 \geq 0 \iff 2x \geq -4 \iff x \geq -2$

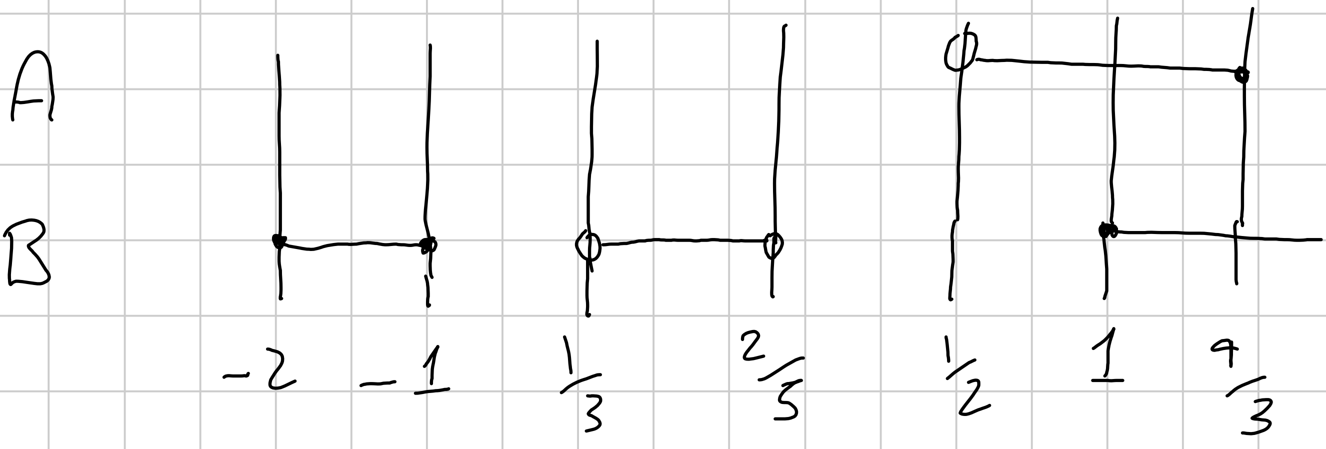
$D_1(x) = 1-3x > 0 \iff x < \frac{1}{3} = \frac{5}{15}$

$D_2(x) = 2-5x > 0 \iff x < \frac{2}{5} = \frac{6}{15}$

		-2		-1		$\frac{1}{3}$		$\frac{2}{5}$		1	
N_1	$++$		$++$		$--$		$--$		$--$		$++$
N_2	$--$		$++$		$++$		$++$		$++$		$++$
D_1	$+$		$+$		$+$		$-$		$-$		$-$
D_2	$+$		$+$		$+$		$+$		$-$		$-$
	$-$		$+$		$-$		$+$		$-$		$+$

$$B = \left\{ x \in \mathbb{R} \mid -2 \leq x \leq -1 \vee \frac{1}{3} < x < \frac{2}{5} \vee x \geq 1 \right\}$$

$$= [-2, -1] \cup \left] \frac{1}{3}, \frac{2}{5} \right[\cup [1, +\infty[$$



$A \cup B = [-2, -1] \cup \left(\frac{1}{3}, \frac{2}{5}\right) \cup \left(\frac{1}{2}, +\infty\right)$

$A \cap B = \left[1, \frac{4}{3}\right]$

$A \setminus B = \left(1/2, 1\right)$

<u>Ex 4</u>

SIA $C = \{ x \in \mathbb{R} \text{ tc } x^4 + 6x^3 + 3x^2 - 10x < 0 \}$

Q:

(i) $\forall x \in \mathbb{R}$ $\quad x \in C \Rightarrow x \leq 1$

(ii) $\forall x \in C$ $\quad (x+2)(x+3) \neq 2$

(iii) $\exists x \in C$ $\quad x^2 > 2$

(iv) $\forall x \in C$ $\quad x^2 > 2$

(v) $\forall x \in C$ $\quad x^2 - 50 < 0$

$P(x) = x^4 + 6x^3 + 3x^2 - 10x < 0 \quad \Leftrightarrow$

$$x \underbrace{\left(x^3 + 6x^2 + 3x - 10\right)}_{q(x)} < 0$$

POSSIBILI RADICI RAZIONALI SONO $\pm 1, \pm 2, \pm 5, \pm 10$

$$q(1) = 1 + 6 + 3 - 10 = 0$$

$(x-1)$ DIVIDE $x^3 + 6x^2 + 3x - 10$

A)

$$\begin{array}{c|ccc|c}
 & 1 & 6 & 3 & -10 \\
1 & & 1 & 7 & 10 \\
\hline
 & 1 & 7 & 10 & 0
\end{array}$$

$$x^3 + 6x^2 + 3x - 10 =$$
$$= (x-1)(x^2 + 7x + 10)$$

B)

$$\begin{array}{l|l}
x^3 + 6x^2 + 3x - 10 & x - 1 \\
\hline
x^3 - x^2 & x^2 + 7x + 10 \\
/\ 7x^2 + 3x & \\
7x^2 - 7x & \\
/\ 10x - 10 & \\
10x - 10 & \\
0 &
\end{array}$$

$$(x^3 + 6x^2 + 3x - 10) =$$
$$= (x-1)(x^2 + 7x + 10)$$

C) $$(x^3 + 6x^2 + 3x - 10) = (x-1)(x^2 + 7x + 10)$$

$$x^2 \cdot (x-1) = x^3 - x^2$$

$$p(x) = x\,q_1(x) = x(x-1)(x^2+7x+10) < 0$$

$$F_1(x) = x \geq 0$$

$$F_2(x) = x-1 \geq 0 \iff x \geq 1$$

$$F_3(x) = x^2+7x+10 \geq 0 \iff$$

$$\Delta = 49-40 = 9 > 0 \quad \begin{cases} x_1 = -2 \\ x_2 = -5 \end{cases}$$

$$\iff x \leq -5 \ \vee \ x \geq -2$$

		-5		-2		0		1	
F_1	$-$		$-$		$-$		$+$		$+$
F_2	$-$		$-$		$-$		$-$		$+$
F_3	$+$		$-$		$+$		$+$		$+$
	$+$		$-$		$+$		$-$		$+$

$$C = (-5, -2) \cup (0, 1)$$

(i) $\forall x \in C \quad x \leq 1$ VERO

(ii) $\forall x \in C \quad (x+2)(x+3) \neq 2$ FALSO!

$(x+2)(x+3)=2 \Leftrightarrow x^2+5x+6-2=0$

$\Leftrightarrow x^2+5x+4=0 \Leftrightarrow (x+4)(x+1)=0$

$\Leftrightarrow x=-4 \vee x=-\underline{1}$

$\exists x \in C$ t.c $(x+2)(x+3)=2$

$x=-4 \in C$

(iii) $\exists x \in C \quad x^2>2$

$(-4)^2=16>2$

(iv) $\forall x \in C \quad x^2>2 \Leftrightarrow$

$\forall x \in C \quad x<-\sqrt{2} \vee x>\sqrt{2}$

$\sqrt{2} \simeq 1,414..$ FALSA

($\neg$) $\exists x \in C$ t.c $-\sqrt{2} \leq x \leq \sqrt{2}$

$x=\frac{1}{2}$

(v) $\forall x \in C \quad x^2 - 50 < 0 \iff$

$\forall x \in C \quad x \in (-\sqrt{50}, \sqrt{50})$ VERA

$5 = \sqrt{25} < \sqrt{50} \iff$

$-\sqrt{50} < -5$

Ex 5 TROVARE LE SOLUZIONI DEL SISTEMA

$$
\begin{cases}
① \ (\sqrt{5}x - 1)(x\sqrt{5} + 1) + 2\left(\dfrac{x}{\sqrt{2}} + 1\right)^2 > 2x^2 + 2 \\
② \ \dfrac{x^2 - x}{3} + \dfrac{x+1}{2} > \dfrac{5x+1}{3} - 1
\end{cases}
$$

$''(5x^2 - \cancel{\sqrt{5}x} + \cancel{\sqrt{5}x} - 1$

$2 = \sqrt{2}\cdot\sqrt{2}$

① $(5x^2 - 1) + 2\left(\dfrac{x^2}{2} + \dfrac{2}{\sqrt{2}}x + 1\right) - 2x^2 - 2 > 0$

$\iff$

$$5x^2 - 1 + x^2 + 2\sqrt{2}x + \cancel{2} - 2x^2 - \cancel{2} > 0$$

$\iff$

$$4x^2 + 2\sqrt{2}x - 1 > 0 \quad ★$$

$\Delta = 8 + 16 = 24 > 0$

$$x_{1,2} = \frac{-2\sqrt{2} \pm \sqrt{24}}{8} \begin{cases} \frac{-2\sqrt{2}-2\sqrt{6}}{8} = -\frac{\sqrt{2}+\sqrt{6}}{4} \\ \frac{-2\sqrt{2}+2\sqrt{6}}{8} = \frac{\sqrt{6}-\sqrt{2}}{4} \end{cases}$$

$$\overset{*}{\Longleftrightarrow} \boxed{x < -\frac{\sqrt{2}+\sqrt{6}}{4} \lor x > \frac{\sqrt{6}-\sqrt{2}}{4}}$$

SOLUZIONE d. 1

2 $$\frac{x^2-x}{3} + \frac{x+1}{2} - \frac{5x+1}{3} + 1 > 0 \Longleftrightarrow$$

$$\Longleftrightarrow \frac{2x^2-2x+3x+3-10x-2+6}{6} > 0 \Longleftrightarrow$$

$$\frac{1}{6}\left(2x^2-9x+7\right) > 0 \Longleftrightarrow$$

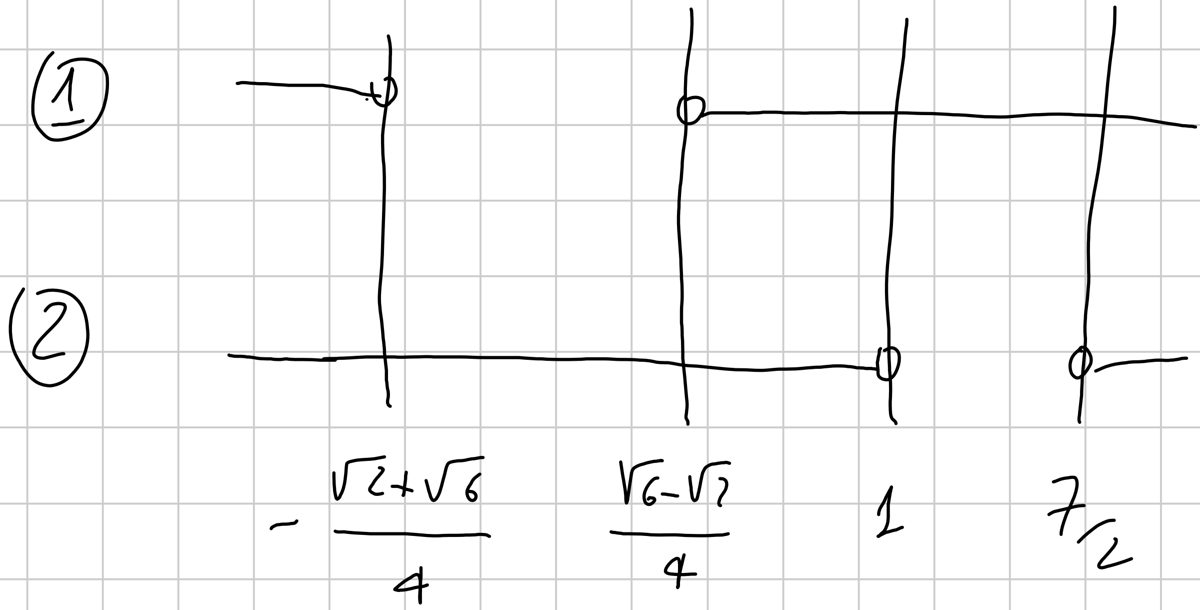
$$2x^2-9x+7 > 0$$

$$\Delta = 81-56 = 25 > 0$$

$$x_{1,2} = \frac{9 \pm 5}{4} \begin{cases} \frac{7}{2} \\ 1 \end{cases}$$

SOLUZIONI DI (2) $\quad x < 1 \ \vee \ x > \frac{7}{2}$

Interseco le soluzioni



$$\frac{\sqrt{6}-\sqrt{2}}{4} < 1 < \frac{7}{2}$$

OSS $\sqrt{6} < 3 = \sqrt{9} \qquad \frac{\sqrt{6}-\sqrt{2}}{4} < \frac{3}{4} < 1$

SOL.NI SISTEMA $\quad x \in \left(-\infty, -\frac{\sqrt{6}+\sqrt{2}}{4}\right) \cup \left(\frac{\sqrt{6}-\sqrt{2}}{4}, 1\right) \cup \left(7/2, +\infty\right)$

ESERCIZI

E1
$$\begin{cases} x^2 - 5x + 6 \geq 0 \\ 2(2x - 9) < x \\ x^2 + 2x - 15 \leq 0 \end{cases}$$

E2

a) $x^3 - 8 < 0$

b) $x^2 + \frac{1}{x^2} + 2 > 0$

c) $(1 - 4x^2)(x^2 - 6x + 7) < 0$

E3

SIA

$$D = \left\{ x \in \mathbb{R} \text{ t.c. } \frac{x^2 + 8x + 4}{x + 1} > 8 \text{ e } x^3 - 8 < 0 \right\}$$

$$Q: \forall x \in D \quad x > 0 \implies x^2 < 0$$

CORREZIONE

E1
$$\begin{cases} x^2 - 5x + 6 \geq 0 & (1) \\ 2(2x - 9) < x & (2) \\ x^2 + 2x - 15 \leq 0 & (3) \end{cases}$$

(1) $x^2 - 5x + 6 \geq 0 \iff$

$(x - 3)(x - 2) \geq 0 \iff$

$x \leq 2 \lor x \geq 3$

(2) $2(2x-9) < x \iff$

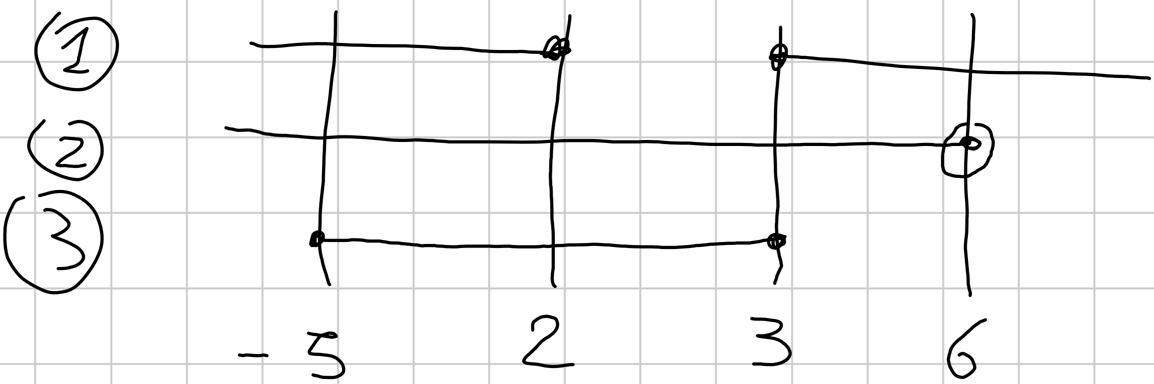
$4x - 18 - x < 0 \iff$

$3x < 18 \iff x < 6$

(3) $x^2 + 2x - 15 \le 0 \iff$

$(x+5)(x-3) \le 0 \iff$

$-5 \le x \le 3$



Sol.ne $x \in [-5, 2] \cup \{3\}$

E2

a) $x^3 - 8 < 0 \iff *$

POSSIBILI RADICI RAZIONALI $\pm 1, \pm 2$
$\pm 4; \pm 8$

$p(x) = x^3 - 8$

$p(1) = -7$
$p(-1) = -9$
$p(2) = 0$ ← RADICE

$$(x^3 - 8) = (x-2)(x^2 + 2x + 4)$$

$*$

$$(x-2)\underbrace{(x^2 + 2x + 4)} < 0 \iff$$

$\Delta < 0, \; a > 0 \Rightarrow x^2 + 2x + 4 > 0 \; \forall x$

$$\iff x < 2$$

b) CE $\boxed{x \neq 0}$

$$x^2 + \frac{1}{x^2} + 2 > 0 \iff \frac{x^4 + 2x^2 + 1}{x^2} > 0$$

$$\iff \frac{(x^2+1)^2}{x^2} > 0 \quad \forall x \in \mathbb{R} \setminus \{0\}$$

c) $(1-4x^2)(x^2-6x+7) < 0 \iff$

- $1-4x^2 \geq 0 \iff 4x^2 - 1 \leq 0$

$\iff -\frac{1}{2} \leq x \leq \frac{1}{2}$

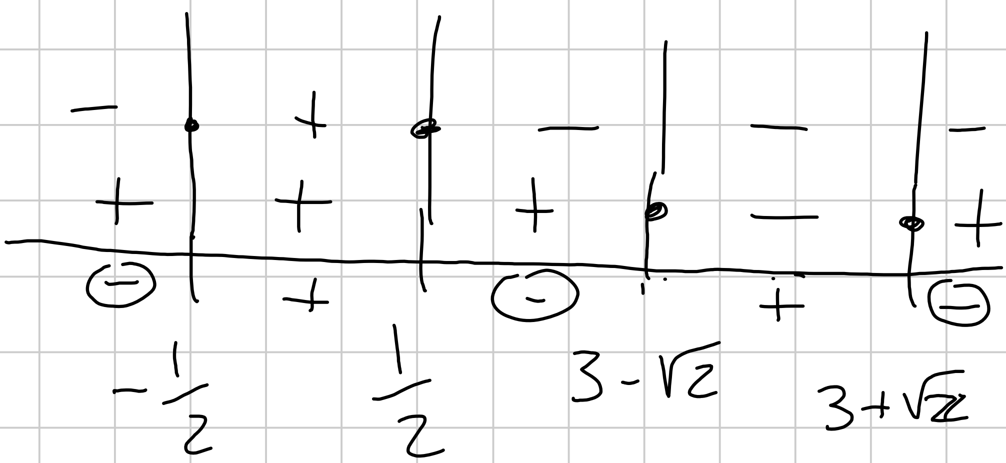
- $(x^2-6x+7) \geq 0 \iff$

$\Delta = 36 - 28 = 8 \quad x_{1,2} = \frac{6 \pm 2\sqrt{2}}{2}$

$x_1 = 3+\sqrt{2}$

$x_2 = 3-\sqrt{2}$

$\iff x \leq 3-\sqrt{2} \vee$
$x \geq 3+\sqrt{2}$



Sol.ne $x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{3}, 3-\sqrt{2}\right)$
$\cup \left(3+\sqrt{2}, +\infty\right)$

E_3 $\quad D = \left\{ x \in \mathbb{R} \text{ t.c. } \dfrac{x^2+8x+4}{x+1} > 8 \right.$ e $\left. x^3 - 8 < 0 \right.$

$Q \quad \forall x \in D \quad x > 0 \Rightarrow x^2 < 0$

(1) $\dfrac{x^2 + \cancel{8x} + 4 - \cancel{8x} - 8}{x+1} > 0 \iff$ $\boxed{\begin{array}{l} CE \\ x \neq -1 \end{array}}$

$\dfrac{x^2 - 4}{x+1} > 0$

$N(x) \geq 0 \iff x \leq -2 \vee x \geq 2$

$D(x) > 0 \iff x > -1$

		-2		-1		2	
N	+	•	−		−	•	+
D	−		−	⊙	+		+
	−		+		−		+

Sol.ne

$x \in (-2, -1) \cup (2, +\infty)$

(2) $x^3 - 8 < 0 \iff x < 2$ (visto sopra)

$D = (-2, -1)$

ALLORA $\forall x \in D \quad x > 0 \Rightarrow x^2 < 0$

E' VERA!

NEGO $\exists x \in D$ t.c. $x > 0$ et $x^2 < 0$

MA NON CI SONO IN D $x > 0$,
QUINDI LA NEGATA E' FALSA (E
QUELLA DI PARTENZA VERA)