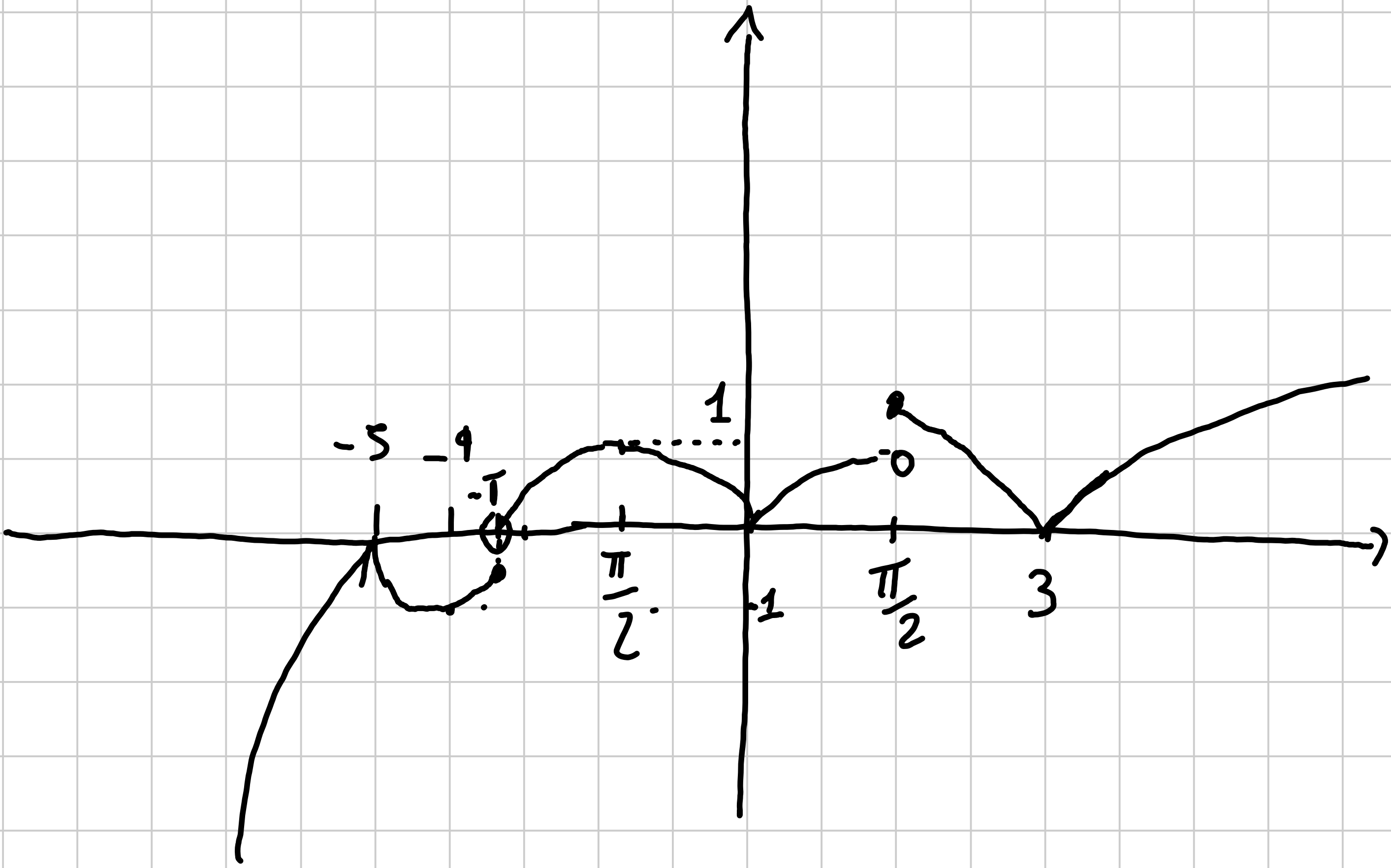
$$f_1 = e^{|x+2|} \qquad f_2 = e^{|x|+2}$$

$$f(x) = e^{|x|}$$

$$f_1 = f(x+2) \qquad f_2 \neq f(x+2)$$

Ex 1

$$f(x) = \begin{cases} -|x^2+8x+15| & x \leq -\pi \\ |\sin|x|| & -\pi < x < \frac{\pi}{2} \\ \sqrt{|x-3|} & x \geq \frac{\pi}{2} \end{cases}$$



$x \leq -\pi$ $\pi \simeq 3,14$

$$f(x) = -|x^2 + 8x + 15|$$

$f(x) \leq 0$ $g(x) = x^2 + 8x + 15$

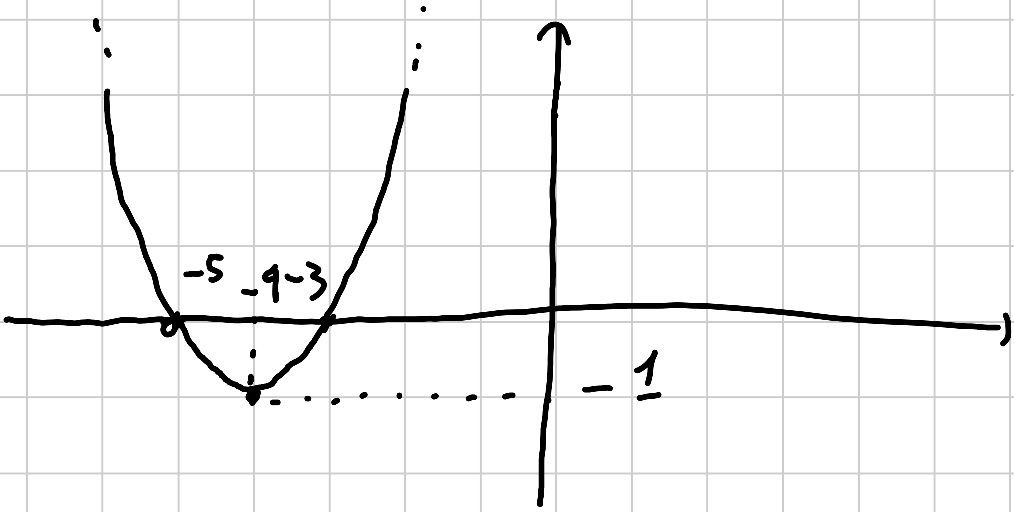
$$g(x) = (x+3)(x+5)$$

$$g(x) = 0 \iff x = -3 \vee x = -5$$

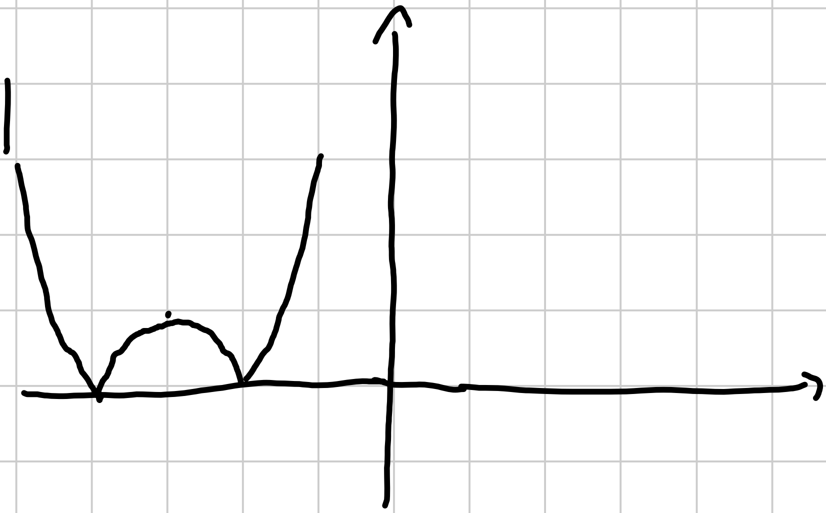
$$g(x) = (x - x_v)^2 + y_v$$

$$g(x) = x^2 + 8x + 15 = (x+4)^2 - 16 + 15$$
$$= (x+4)^2 - 1 = (x-(-4))^2 - 1$$

$V: (-4, -1)$



$|g(x)|$



$-|g(x)|$

$$-\pi < x < \frac{\pi}{2}$$

$$f(x) = |\sin|x|| \qquad f(x) \geq 0$$

$$-\pi < x \leq 0$$

$$f(x) = |\sin(-x)| = $$

$$= |-\sin(x)| =$$

$$= \underline{|\sin x|}$$

$$0 < x < \frac{\pi}{2}$$

$$f(x) = \underline{|\operatorname{sen} x|}$$

$$-\pi < x < \frac{\pi}{2} \qquad f(x) = |\sin x|$$

$$f(-\pi) = 0 \qquad f\left(-\frac{\pi}{2}\right) = 1$$

$$f(0) = 0 \qquad f\left(\frac{\pi}{2}\right) = 1$$

$x > \pi/2$

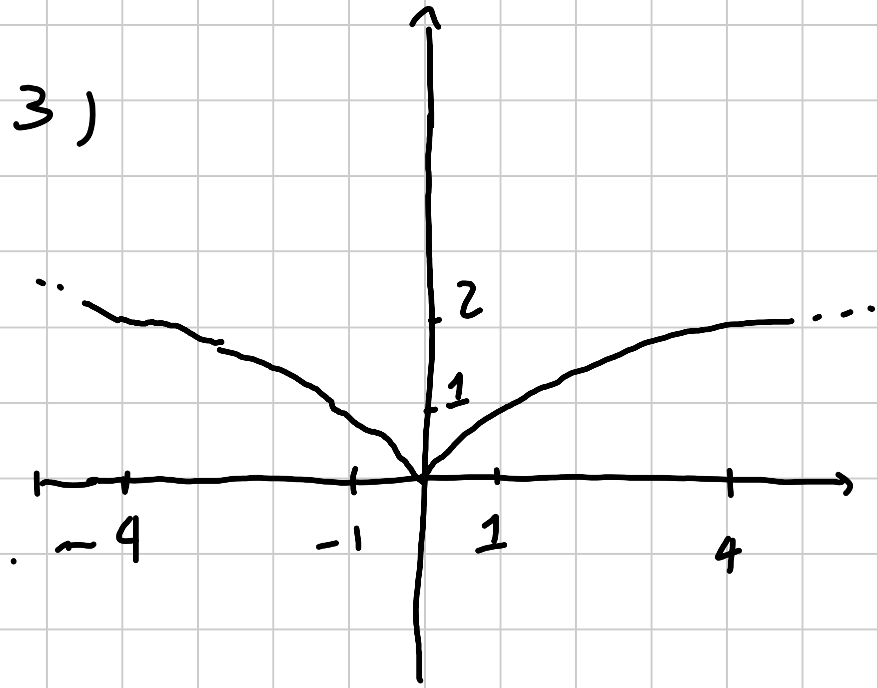
$$f(x) = \sqrt{|x-3|}$$

" NO CONDIZ. AGG.

$|x-3| \geq 0$ "

$$g(x) = \sqrt{|x|}$$

$$f(x) = g(x-3)$$



$$|f(x)| = \frac{1}{2} \iff f(x) = \frac{1}{2}$$

$$\vee \quad f(x) = -\frac{1}{2}$$

$\boxed{x \leq -\pi}$ oss $f(x) \leq 0$

$$f(x) = -\frac{1}{2} \iff -\left|x^2+8x+15\right| = -\frac{1}{2}$$

$$\iff \left|x^2+8x+15\right| = \frac{1}{2} \iff$$

$$1 \begin{cases} x \leq -\pi \\ x^2+8x+15 = \frac{1}{2} \\ x^2+8x+15 \geq 0 \end{cases} \vee \quad 2 \begin{cases} x \leq -\pi \\ x^2+8x+15 = -\frac{1}{2} \end{cases}$$

$$1 \begin{cases} x \leq -\pi \\ 2x^2+16x+29 = 0 \end{cases} \vee \quad 2 \begin{cases} x \leq -\pi \\ 2x^2+16x+31 = 0 \end{cases}$$

(....)

1 Sol

$$\begin{cases} x = \frac{-8-\sqrt{2}}{2} \quad \vee \quad \text{ok} \\ x = -\frac{8+\sqrt{2}}{2} \quad \boxed{\text{ok}} \\ x \leq -\pi \end{cases}$$

$\sqrt{2} \sim 1{,}4$

$$\frac{-8+\sqrt{2}}{2} \sim \frac{6{,}6}{2} = 3{,}3$$

(2) Sol $\begin{cases} x = -4-\sqrt{2} \vee x = -4+\sqrt{2} \\ x \leq -\pi \end{cases}$

$x = -4-\sqrt{2} \mid \boxed{ok}$

$x = -4+\sqrt{2} \sim -4+1,4 = -2,6$

$> -\pi$

$-\pi < x < \frac{\pi}{2}$

$$f(x) = |\sin x| = \frac{1}{2} \iff$$

$$\sin x = \frac{1}{2} \iff x \in \left\{\frac{\pi}{6}\right\}$$

$$\vee \sin x = -\frac{1}{2} \iff x \in \left\{-\frac{\pi}{6}, -\frac{5}{6}\pi\right\}$$

$x \geq \frac{\pi}{2}$

$$f(x) = \sqrt{|x-3|} = \frac{1}{2} \iff$$

$$\iff |x-3| = \frac{1}{4} \iff$$

$$\iff \begin{cases} x-3 = \frac{1}{4} \vee x-3 = -\frac{1}{4} \\ x \geq \frac{\pi}{2} \end{cases}$$

$$\iff \begin{cases} x = 13/4 \vee x = \frac{11}{4} \quad \boxed{\text{ok}} \\ x \geq \frac{\pi}{2} \end{cases}$$

EX 2 SOLUZ. IN $\mathbb{R}$ DI

$$|x^2 + x - 2| < 3x + 1 \iff$$

$$-(3x+1) < x^2+x-2 < 3x+1$$

$$\Leftrightarrow \begin{cases} x^2+x-2 > -3x-1 \\ x^2+x-2 < 3x+1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2+4x-1 > 0 \\ x^2-2x-3 < 0 \Leftrightarrow (x-3)(x+1) < 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \quad x^2+4x-1 = 0 \Leftrightarrow x_{1,2} = \frac{-4 \pm \sqrt{16+4}}{2}$$

$$x_{1,2} = -2 \pm \sqrt{5}$$

$$\begin{cases} x < -2-\sqrt{5} \;\vee\; x > -2+\sqrt{5} \\ -1 < x < 3 \end{cases}$$

Soluz. $x \in \left(-2+\sqrt{5}\, ,\, 3\right)$

Ex 3 TROVARE LE SOLUZ. $x \in [0, 2\pi)$

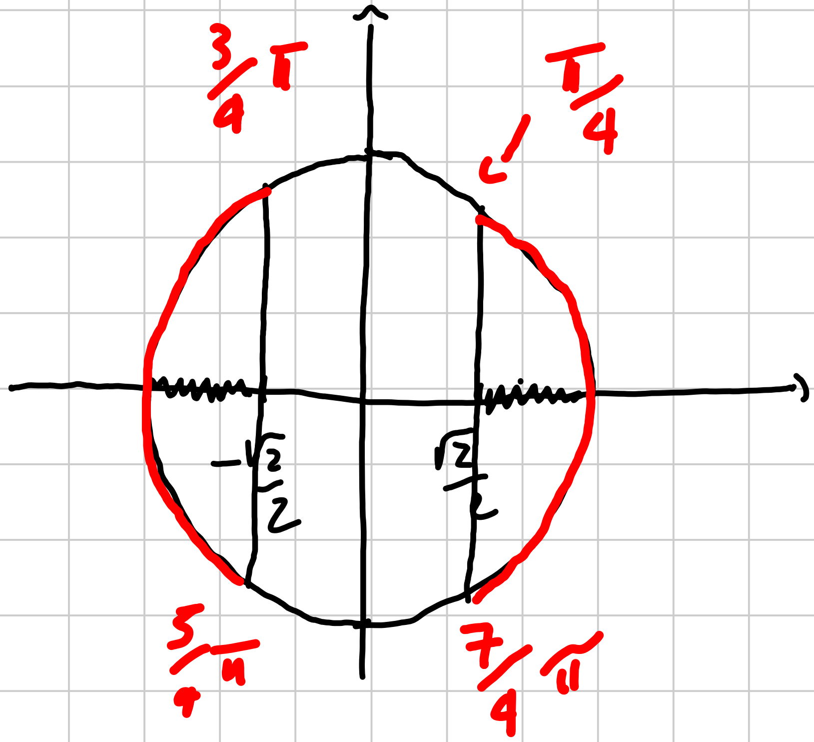
$$\underbrace{\left(\cos^2 x - \frac{1}{2}\right)}_{F_1}\underbrace{\left(\frac{1}{4} - \operatorname{sen}^2 x\right)}_{F_2} > 0$$

$$F_1(x) \geq 0 \iff \cos^2 x - \frac{1}{2} \geq 0 \iff$$

$$\cos^2 x \geq \frac{1}{2} \iff$$

$$\cos x \leq -\sqrt{\frac{1}{2}} \vee \cos x \geq \sqrt{\frac{1}{2}}$$

$$\Leftrightarrow \quad \cos x < -\frac{\sqrt{2}}{2} \quad \vee \quad \cos x > \frac{\sqrt{2}}{2}$$



Solut $\quad x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3}{4}\pi, \frac{5}{4}\pi\right] \cup$

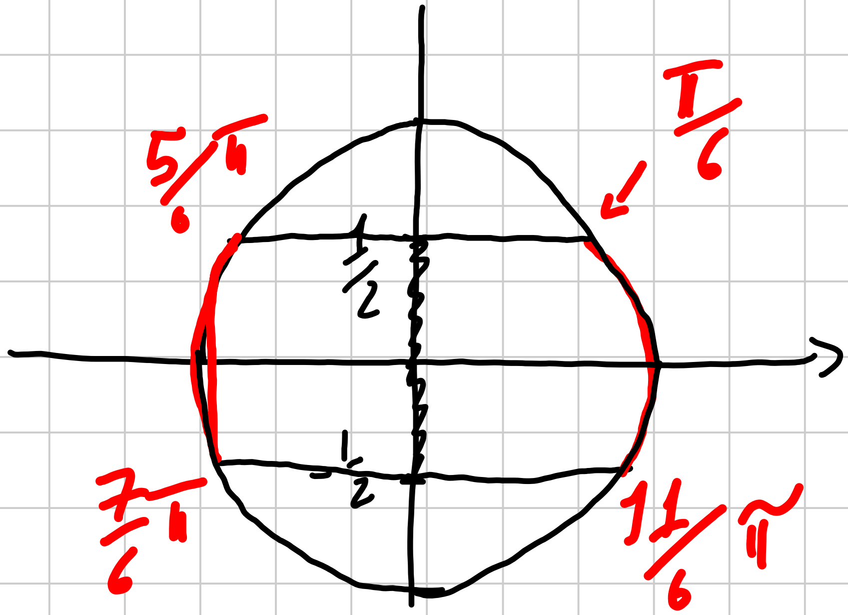
$$\cup \left[\frac{7}{4}\pi, 2\pi\right)$$

2

$$\frac{1}{4} - \sin^2 x \geq 0 \quad \Leftrightarrow$$

$$\sin^2 x \leq \frac{1}{4} \quad \Leftrightarrow$$

$$-\frac{1}{2} < \sin x < \frac{1}{2}$$



$$\text{Sol}^{②} \Rightarrow x \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5}{6}\pi, \frac{7}{6}\pi\right] \cup \left[\frac{11}{6}\pi, 2\pi\right)$$

	0 – $\frac{\pi}{6}$	$\frac{\pi}{6}$ – $\frac{\pi}{4}$	$\frac{\pi}{4}$ – $\frac{3}{4}\pi$	$\frac{3}{4}\pi$ – $\frac{5}{6}\pi$	$\frac{5}{6}\pi$ – $\frac{7}{6}\pi$	$\frac{7}{6}\pi$ – $\frac{5}{4}\pi$	$\frac{5}{4}\pi$ – $\frac{7}{4}\pi$	$\frac{7}{4}\pi$ – $\frac{11}{6}\pi$	$\frac{11}{6}\pi$ – 2π
$F_1(x)$	+	+	-	+	+	+	-	+	+
$F_2(x)$	+	-	-	-	+	-	-	-	+
$F_1 F_2$	+	-	+	-	+	-	+	-	+

Solu? $x \in \left[0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{4}, \frac{3}{4}\pi\right) \cup \ldots$

Ex 4 Soluz $x \in \mathbb{R}$

$$\frac{3^{x+1}}{27^{2x}} < \frac{1}{3^{x^2+5}} \iff$$

$$\frac{3^{x+1}}{3^{6x}} < \frac{1}{3^{x^2+5}} \iff 3^{x+1-6x} < 3^{-(x^2+5)}$$

$$\iff (x+1-6x) < -(x^2+5) \iff$$

$$x^2+5-5x+1 < 0 \iff$$

$$x^2-5x+6 = (x-2)(x-3) < 0$$

$\Leftrightarrow \quad 2 < x < 3$

EX

$$2^{x+1} \geq 5^{1-x} \Leftrightarrow$$ APPLICO log

$$\log 2^{x+1} \geq \log 5^{1-x} \Leftrightarrow$$

$$(x+1)\log 2 \geq (1-x)\log 5 \Leftrightarrow$$

$$x(\log 2 + \log 5) \geq \log 5 - \log 2$$

$$x \log 10 \geq \log \frac{5}{2} \Leftrightarrow$$

$$x \geq \frac{\log \frac{5}{2}}{\log 10} \left(= \log_{10} \frac{5}{2} \right)$$

Ex

CE $x>0$

$$\log_2 x^2 - 2\log_4 x + 1 > 0 \iff$$

$$2\log_2 x - \cancel{2}\,\frac{\log_2 x}{\log_2 4} + 1 > 0 \iff$$

$$\log_2 4 = \cancel{2}$$

$$2\log_2 x - \log_2 x + 1 > 0 \iff$$

$$\log_2 x > -1 = \log_2 2^{-1}$$

$$\iff$$

$$x > \frac{1}{2}$$

Ex

$$\log_3\left(\log_3(2x-5)\right) < 0 = \log_3 1$$

$\iff$

$$\log_3(2x-5) < 1 = \log_3 3$$

$\iff$

$$2x-5 < 3 \iff$$

$$x < 4$$

CE

$$\begin{cases} 2x-5>0 \\ \log_3(2x-5)>0 \end{cases}$$

$$\begin{cases} 2x-5>0 \\ 2x-5>1 \end{cases}$$

$$\boxed{x>3}$$

Soln. $x \in (3, 4)$

$$\begin{cases} \log_2(x-1) > 1 \\ \log_{\frac{1}{2}} x > 2 \end{cases}$$

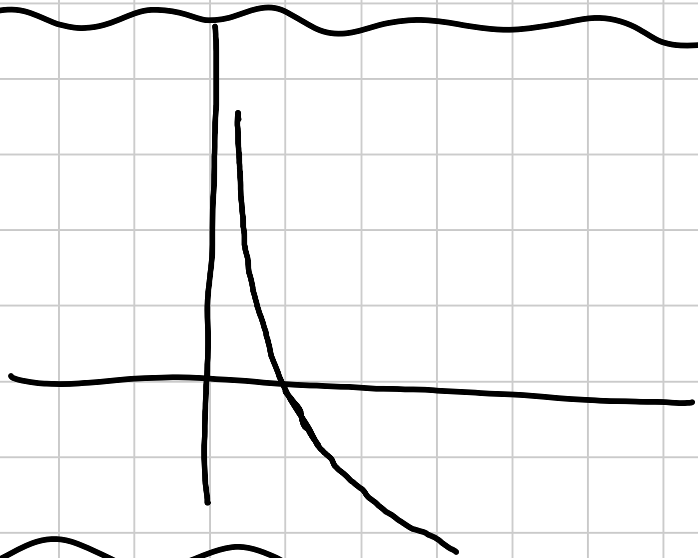
CE

$$\begin{cases} x>1 \\ x>0 \end{cases}$$

$\hookrightarrow x > 1$

$$\Longleftrightarrow \begin{cases} \log_2 (x-1) > \log_2 2 \Longleftrightarrow x-1>2 \\ \qquad\qquad\qquad\qquad\qquad\qquad \Longleftrightarrow x > 3 \\ \log_{\frac{1}{2}} x > 2 = \log_{\frac{1}{2}} \frac{1}{4} \Longleftrightarrow x < \frac{1}{4} \end{cases}$$

$\log_a x \quad 0<a<1$



IL SISTEMA NON HA SOLUZIONI!

Ex

1. $y = -\frac{1}{2}x^2 - \frac{3}{2}x + 5 =$

$$= -\frac{1}{2}\left(x^2 + 3x\right) + 5 =$$

$$= -\frac{1}{2}\left(\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}\right) + 5 =$$

$$= -\frac{1}{2}\left(x+\frac{3}{2}\right)^2 + \underbrace{5 + \frac{9}{8}}_{\frac{49}{8}}$$

$$V : \left(-\frac{3}{2}, \frac{49}{8}\right)$$

2. $$5x^2 + 5y^2 - 14x = 0$$

$$\Leftrightarrow$$

$$5x^2 - 14x + 5y^2 = 0$$

$$\Leftrightarrow$$

$$5\left(x^2 - \frac{14}{5}x\right) + 5y^2 = 0$$

$$\Leftrightarrow$$

$$5\left(\left(x - \frac{7}{5}\right)^2 - \frac{49}{25}\right) + 5y^2 = 0$$

$$\Leftrightarrow$$

$$5\left(x-\frac{7}{5}\right)^2-\frac{49}{5}+5y^2=0$$

$$\left(x-\frac{7}{5}\right)^2+y^2=\frac{49}{25}$$

$$C:\left(\frac{7}{5},0\right) \quad R=7/5$$

3. $$\frac{1}{2}x^2+\frac{1}{16}y^2=x-\frac{1}{4}y-\frac{1}{4}$$

$$\frac{1}{2}\left(x^2-2x\right)+\frac{1}{16}\left(y^2+4y\right)=-\frac{1}{4}$$

$$\frac{1}{2}\left(\left(x-1\right)^2-1\right)+\frac{1}{16}\left(\left(y+2\right)^2-4\right)=-\frac{1}{4}$$

$$\Leftrightarrow \quad \frac{1}{2}(x-1)^2-\frac{1}{2}+\frac{1}{16}(y+2)^2-\cancel{\frac{1}{4}}=-\cancel{\frac{1}{4}}$$

$$\frac{1}{2}(x-1)^2 + \frac{1}{16}(y+2)^2 = \frac{1}{2} \iff$$

$$(x-1)^2 + \frac{1}{8}(y+2)^2 = 1$$

$C:(1,-2)$ $\quad a=1$

$b = 2\sqrt{2}$

$$\frac{1}{3}x^2 + y^2 - 2x - 2y + 5 = 0 \iff$$

$$\frac{1}{3}\left(x^2 - 6x\right) + \left(y^2 - 2y\right) + 5 = 0$$

$\iff$

$$\frac{1}{3}\left((x-3)^2 - 9\right) + (y-1)^2 - 1 + 5 = 0$$

$\iff$

$$\frac{1}{3}(x-3)^2 - 3 + (y-1)^2 - 1 + 5 = 0$$

$$\frac{1}{3}(x-3)^2 + (y-1)^2 = -1$$

$$\{(x,y) \in \mathbb{R}^2 \text{ t.c. } \ldots\ldots = 0\} = \emptyset$$

$$xy = 3 \qquad -3$$

IPERBOLE